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A NEUTRAL LINE DISCHARGE THEORY OF THE AURORA POLARIS

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CONTENTS

A theory of the aurora polaris is proposed which attempts to explain many features of the complicated morphology of auroral displays. One basis of the theory is the presence, during magnetic disturbance, of additional or enhanced magnetic fields due to electric currents within a distance of several earth radii from the earth's centre. One such field (denoted by DCF) is due to electric currents flowing near the inner surface of the solar stream that then envelopes the earth. A hollow is carved in the stream by the geomagnetic field. The other field (denoted by DR) is that of an electric ring current, additional or enhanced, that flows westward round the earth. This is carried by the particles of the Van Allen belts. A third field (denoted by DP) is that of the disturbance currents that flow in the ionosphere, under the impulsion of electromotive forces generated mainly in polar regions.

We consider it likely that during magnetic storms and auroral displays, neutral lines appear in the magnetic field near the earth. These will lie mainly on the dark side of the earth, in or near the equatorial plane, on the nearer side of the ring current. At times these lines may extend over more than 180° of longitude, so that a part of them may lie on the sunward side of the earth. These neutral lines are of two types, which we call O and X; they appear together, in pairs. During disturbed conditions there may be more than one pair. Lines of force cross at points on X neutral lines, but they do not pass through O neutral lines.

As Dungey has shown, charged particles will tend to be concentrated near X points (of which the X neutral lines are the locus). Charges drawn toward the neutral line will be discharged into the earth's atmosphere along the lines of magnetic force. We suggest that the location, nature and motions of the auroral forms are determined by the position, form and motion of the X neutral lines, lying in or near the plane of the geomagnetic equator.

It seems necessary to suppose, in addition, that an electric field arises sporadically along the X lines. When this is absent, the aurora appears as a quiet arc. The onset of the suggested electric field concentrates the charges more narrowly near the X line and near the lines of force that extend from it to the auroral zone. This produces extremely thin-rayed auroral arcs.

The above concentration of electrons near an X neutral line produces a large flux of electrons, while the proton flux is diminished. A dynamical instability due to this flux difference (the space charge density is supposed to be very small) produces a slight separation of protons and electrons along and near the lines of force through the X line. Hence in the auroral ionosphere there is an associated electric field. This is usually directed towards the equator. It drives electric current, usually westward, along the auroral zones, and produces the strong magnetic disturbances (DP) there observed. Birkeland called these polar elementary storms.

The rapid auroral changes are ascribed to instabilities of the magnetic field in the region near the X line or lines, to the rear of the earth, where the resultant magnetic field is weak. The ray structure in the auroral arc is ascribed to an instability of the thin sheet of electron flow.

Cosmic rockets have shown that the magnetic field, up to and beyond ten earth radii, departs from the values corresponding to the internally produced main geomagnetic field. As yet these explorations do not seem to have disclosed the existence of *reversals* of the field in or near the magnetic equatorial plane. But on the basis of our auroral hypothesis, we predict with considerable confidence that such reversals will be found to occur, on the dark side of the earth, during great auroral displays.

The theory here proposed is discussed in connexion with recent I.G.Y. and I.G.C. auroral, magnetic and other data.

1. Introduction

The aurora polaris, which provides wonderful displays in the polar night sky, offers some of the most difficult problems in geophysics. Many partial theories have been put forward to explain one or more auroral phenomena, but all are inevitably far from complete. Störmer (1955) and Chamberlain (1958) have given good reviews of these theories. In this section, therefore, mention will be made only of such recent theories as have some relation to the new theory given in this paper.

Birkeland's *terrella* experiments led the way to the modern theories. He studied the motions of electrons in a dipole field, which later were mathematically discussed by Störmer. Störmer tried to associate the motions of solitary charged particles in the earth's field with various features of the aurora. He found that certain regions around the earth were inaccessible ('forbidden') to the particles, while others were 'allowed'. His theory was criticized on the ground that the mutual repulsion of the particles (all of one sign) would disperse the stream before it could reach the earth. To explain the movement of the aurora to lower latitudes during great magnetic storms, he suggested that many of the particles sweeping partly round the earth (at a distance comparable with that of the moon) might have a magnetic effect equivalent to that of a complete westward electric current—now generally called a ring current in this connexion. However, as was pointed out by Chapman & Ferraro (1933), his ring current would greatly change the earth's field in its locality, so that the calculation on which its presence was postulated would not be valid.

Chapman & Ferraro (1931 *a*, *b*) were the first to infer that during magnetic storms the important interaction of solar particles with the earth's magnetic field will occur much nearer to the earth, less than 10 earth radii. The development of their magnetic storm theory was based on the hypothesis, proposed by Lindemann (1919), of a neutral ionized stream. This condition, that the number of positive and negative particles must be nearly equal, is one of the most fundamental in plasma physics. Chapman & Ferraro criticized Störmer's theory on the ground that in a stream of particles, unless they are of extremely high energy and low number density, such as cosmic-ray particles, the oppositely charged particles must affect each other's motion by their Coulomb field. They showed (1940) that if the density was low enough for the particles to travel as Störmer had calculated, it would be far too low to produce appreciable magnetic effects.

Later Alfvén (1940, 1950) advanced one of the most fundamental concepts in modern plasma physics, that of the 'guiding centre'. This relates to low-energy particles in a strong magnetic field. The particle gyrating in a strong field is replaced by a small magnet with moment μ which is invariant. Alfvén showed the usefulness of his simple 'smoothed out' version of the path of such a gyrating particle by comparing it with Störmer's complicated path. He considered the case of a particle rapidly oscillating around and along the lines of magnetic force in a dipole field, with slow drift motion in longitude.

Chapman & Ferraro (1931 *b*) suggested that particles might be 'trapped' from the surface of a hollow in the solar stream, at a distance of several earth radii beyond the earth away from the sun. This speculation remains unconfirmed and unrefuted. Singer (1957), with notable insight, identified the motion of the trapped particles with that considered by Alfvén. This led him to predict a belt of trapped particles in the earth's field, having the form of the

Van Allen radiation belts, before these were discovered by the satellite researches (Van Allen & Frank 1959). The Van Allen radiation belts are associated with a varying electric ring current at several earth radii. This ring current has been calculated by Dessler & Parker (1959), by Akasofu (1960*a*) and by ourselves (1961*c*, *d*).

Plasma theory predicted another important invariant of the motion of a particle in a magnetic field, besides its equivalent magnetic moment μ. This conception was successfully applied to the motions of the electrons of high energy (of order a few MeV) produced by the Argus Experiment (Van Allen, McIlwain & Ludwig 1959). The electron shell which enclosed the earth at that time was located exactly as predicted by the second invariant law.

This led Vestine & Sibley (1959) to produce 'Calculated isochasms', which agree well with the actual isochasms first given by Fritz (1874), and revised later by Vestine (1944). This confirms that the motions of the auroral particles in the earth's field accord with the constancy of the two invariants.

However, as indicated by Chapman & Ferraro in 1931, these particles move in their collective Coulomb field. Plasma theory discusses the macroscopic behaviour of auroral particles by summing the individual motions of the particles, defined by the two invariants, under the condition of electrical neutrality. Any slight charge separation will produce a strong electrostatic field between them, and their motions cannot be independent.

Meanwhile quite another line of development was followed by Hoyle (1949, p. 102) and later by Dungey (1953). They assumed that part of the solar magnetic field is carried away by solar streams, and that at times the earth's dipole field is immersed in a rather uniform solar field. They concluded that two *neutral points* arise on opposite sides of the earth, and they suggested that auroral particles would there be accelerated. But it seems to us unlikely that two such neutral points would suffice to explain the complicated auroral morphology.

Alfvén (1939, 1950, 1955) and Martyn (1951) also put forward auroral theories, although they did not specifically discuss the detailed morphology of auroras. Alfvén's theory is based on the postulated presence of a general electric field enveloping the earth. The motions of the charged particles are inferred to be such as to produce a charge accumulation in the equatorial plane, whence the particles find their way towards the earth along the lines of force.

Martyn based his ideas on the hypothetical polarized ring current discussed by Chapman & Ferraro (1933), located at several earth radii. As they indicated, charged particles are expelled from the inner and outer sides of the surface. Martyn inferred that this produces electric polarization in the ionosphere, which impels the electric current flow there.

In this paper we develop an auroral theory along new lines. We think it is likely that the onset of a solar stream and the growth of the ring current will produce *neutral lines* in the magnetic field, at several earth radii. We suggest that these neutral lines are the proximate sources of the particles that produce auroral displays. It seems to us that the major auroral phenomena may be explicable as the direct or indirect result of certain complicated electromagnetic changes, which we indicate, that may occur in or near the earth's geomagnetic equatorial plane. We show how such geomagnetic field changes may account for many features of the complicated morphology of auroral displays. Lack of certain detailed information, and some mathematical difficulties, prevent our presentation of a more complete theory. But we hope to develop our ideas further in later papers.

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2. The DM, DCF, DR, DP and DSM fields

The discovery of the Van Allen 'radiation' belts seems to confirm a long-standing tentative 'geomagnetic' inference (Schmidt 1917) that a westward electric current permanently encircles the earth—waxing and waning irregularly, in association with solar and geomagnetic activity.

Solar streams and clouds of neutral ionized gas impinge on the earth from time to time. The geomagnetic field resists their advance, and carves a hollow in them. The 'vertex' of the hollow is on the sunward side of the earth. On the dark side the hollow may be open, or if it has an end, this may be far away. Near the surface of the hollow the positive and negative charges are differently deflected sideways. Their motion corresponds to electric current flow over the surface of the hollow, mostly on the sunward side. The current flow is substantially confined to a thin layer near the surface. Many of the deflected particles are turned backwards or sideways into the stream, away from the surface (Chapman & Ferraro 1931 *a*, *b*; 1940; Ferraro 1952).

However, in some way not yet understood, some of the solar gas is not turned away, but becomes 'trapped' in the geomagnetic field. The outer belt, at least, seems likely to be maintained by such additions (Van Allen 1959). They irregularly replace the continual loss of particles from the belt. Moreover, some of the solar gas quickly finds its way into the earth's atmosphere, especially into the auroral zones. There electric currents are set up in the polar ionosphere. They spread over the polar caps and over the great middle belt of the ionosphere between the two zones.

These are the opening events in magnetic storms. The onrush of the front of the solar stream may be very rapidly halted—within a minute or so. During this time the electric currents near its hollow surface are set up. Worldwide ionospheric electric currents driven by electric forces in the auroral zones may be produced with like suddenness. Great magnetic storms typically begin with such a sudden commencement.

At magnetically quiet times the magnetic state of the earth and its near surroundings seems to be as follows. There is the main field M, proceeding from within. In the ionosphere, at a height of order 100 km, there are electric current systems (Sq and L) whose magnetic fields at the earth's surface produce the solar daily and lunar daily magnetic changes there observed. These currents are stronger over the sunlit than over the dark hemisphere. They are intensified—in what is called an electrojet—along a narrow zone lying along the magnetic equator. Further, there is the field R of the ring currents in the Van Allen belts. The onset of a solar stream or cloud brings additional fields into play. The additional fields may all be characterized by the letter D, signifying magnetic disturbance. First, there is the field of the currents over the hollow surface of the solar gas. This may be denoted by DCF; the letters CF refer to the *corpuscular flux*. The field DCF is set up suddenly, and is maintained as long as the corpuscular flux continues. The flux is likely to have irregular variations. These will change the form and size of the hollow, and also the DCF current system and magnetic field.

A second additional field is DR, signifying the *change* in the field of the ring current. The very rapid growth of the DCF field seems likely to produce *some* change in the field R at the onset of the storm. But the initial DR field seems likely to be small. However, later, over a period of several hours, the DR field grows and becomes much greater than the DCF field. During the last phase of the storm the DR field dies away. Its decay is most rapid, the greater its maximum intensity. The rate of decay later decreases.

The third additional field may be denoted by DP. The letter P here signifies *polar*. The symbol DP is used because this field and the currents that produce it are strongest in the polar regions, especially near the auroral zone. The currents are driven by electromotive forces rapidly set up there. Thence the currents spread all over the earth, so that the DP field is world wide. Produced in the ionosphere, it spreads also to the space inside and outside the earth. But the linear scale of its external influence will be less than that of the main field M, because its spherical harmonic components are of higher degree than those of the field M.

There may be a fourth addition to the pre-existing fields during the storm. The solar gas may carry with it a magnetic field transported away from the sun. This field may be denoted by DSM (disturbing solar magnetism). As yet little is known about such fields.

Thus, in all, the disturbing field D during a magnetic storm may be divided into the following four main parts.

$$D = DCF + DR + DP + DSM.$$

The four parts have different characters and variations. The field DCF *increases* the field at the earth's surface in low latitudes—more so over the sunlit than over the dark hemisphere, as the DCF currents are mainly on the sunward side of the earth. The DCF field may fluctuate during the continuance of the onflow of the solar stream, as the intensity of the stream varies. The flux may end either because the solar gas has the form of a cloud, of extent limited in every direction, or because, although it is a continuing stream, the sun's rotation gradually sweeps the onflow forward, beyond the more slowly orbiting earth.

The DR field *reduces* the earth's surface field in low latitudes. It is likely to be the same, or nearly so, all round the earth. This field gradually grows and then declines.

The DP field has a more complicated distribution. Like the Sq and L fields, it is amplified along the magnetic equator. But it is strongest near the auroral zone and over the polar cap. This current system and field often fluctuate greatly during a magnetic storm. The sudden bursts of intensity of the DP field, which Birkeland (1908) called polar elementary storms, seem to be associated with the most active manifestations of the aurora. They are also associated with large increases of absorption of cosmic radio noise along the auroral zones, and with a notable influx of electrons into the auroral ionosphere—as revealed by cosmic ray recording instruments carried on balloons or rockets (Van Allen 1957; Winckler 1960).

Very little can yet be said about the DSM field and its changes. In the following discussion its possible presence will mostly be disregarded. The ring current is an accompaniment of the somewhat complex motions of the energetic protons and electrons in the radiation belts. These motions are governed by the total magnetic field. The chief control is always by the main field M, but at the location of the ring current its own field may also have a significant influence on the motions of the particles. During magnetic storms the DCF field also will modify the normal system of particle motions. The influence upon them exerted by the Sq and L fields, and probably also by the DP field, will be of minor importance. The role of the DSM field in this connexion is at present quite uncertain.

Presumably the belts and ring current are ordinarily symmetrical about the earth's dipole axis (apart from slight irregularities due to the non-dipole field). The DCF field imparts a notable asymmetry to the total field, though this asymmetry is not very marked at the earth's surface. The image dipole conception introduced by Chapman & Ferraro (1931 *a*, *b*) indicates crudely how the DCF field decreases within the hollow of the solar stream along the line through the centres of the sun and earth.

In this conception the stream surface is idealized by being considered as a plane, normal to the sun–earth line. The image point C' of the earth's centre C, mirrored in this plane, is on the sunward side of the plane; C and C' are at equal distances from the plane. The DCF field on the earthward side of the plane is (roughly) that of a dipole at C', equal in moment and direction to the earth's equivalent point dipole. The plane may be taken to pass through the vertex of the (actually curved surface of the) hollow. Thus the DCF field intensity decreases with increasing distance from the vertex.

3. Motions of charged particles in the earth's magnetic field

(*a*) *Drift motion*

In order to study in detail the behaviour of auroral particles, the fundamental equations are briefly reviewed and discussed in this section (cf. Alfvén 1950, chap. II; Spitzer 1952; Watson 1956; Parker 1957). In a non-uniform magnetic field the motions of charged particles are extremely complicated. But in a sufficiently strong magnetic field we can regard the particle motion as being a spiral motion about a 'guiding centre' which slowly drifts. This representation by a 'guiding centre' is applicable under the following conditions.

(1) The average radius of gyration is much smaller than the scale length of the system considered.

(2) The period of gyration is much shorter than the other scale times associated with the phenomena.

(3) The intensity of the electric field, if any, is small. The increase of energy and the drift speed of particles due to the electric field are much less than their original energy and speed, respectively.

Chapman (1960) defines the scale length L and the scale time T of a vector function $\boldsymbol{F}$ of position as follows

$$L = F/(W\colon \overline{W})^{\frac{1}{2}} = F/(W_{\alpha\beta}\overline{W}_{\beta\alpha})^{\frac{1}{2}}, \tag{1}$$

$$1/T = \mathrm{d}(\ln F)/\mathrm{d}t. \tag{2}$$

Here W and $\overline{W}$ denote $\nabla \boldsymbol{F}$ and its conjugate dyadic, respectively. The following symbols refer to the charged particle and the magnetic field

c = the speed of light,
$\boldsymbol{H}$ = the intensity of the magnetic field,
m = the mass of the particle,
m_0 = the rest mass of the particle,
e = its charge ($= 4{\cdot}802 \times 10^{-10}$ e.s.u.),
$\boldsymbol{w}$ = its velocity,
θ = the angle between $\boldsymbol{w}$ and $\boldsymbol{H}$,
w_s, w_n = the components of $\boldsymbol{w}$ along and perpendicular to $\boldsymbol{H}$.

Thus $$w_s = w\cos\theta, \quad w_n = w\sin\theta, \quad w^2 = w_s^2 + w_n^2.$$

The radius of gyration R is given by

$$R = cmw_n/eH. \tag{3}$$

The maximum radius of gyration will be

$$R = cmw/eH. \tag{4}$$

The relativistic form of (4) is given by Livingston (1954, p. 20). It is

$$R = \frac{\{K(K+2E_0)\}^{\frac{1}{2}}}{eH}, \tag{5}$$

where K = the kinetic energy of the particle

$$(= \tfrac{1}{2}m_0 w^2 + \tfrac{3}{8}m_0(w^4/c^2) + \ldots),$$

E_0 = the rest energy $= m_0 c^2$.

We use the subscripts p and e for protons and electrons, respectively. Then

$$m_{0p}c^2 = 1\cdot506\times10^{-3}\,\text{erg} = 940\cdot1\,\text{MeV},$$

$$m_{0e}c^2 = 8\cdot196\times10^{-7}\,\text{erg} = 0\cdot5116\,\text{MeV}.$$

The period of gyration P is given by

$$P = f_0^{-1}(1+K/E_0), \tag{6}$$

where

$$f_0 = eH/2\pi m_0 c. \tag{7}$$

The energies of typical auroral protons and electrons are thought to be $K_p = 130$ keV, $K_e = 30$ keV, respectively. In the case of protons K is much less than E_0, but for electrons K/E_0 is appreciable. However, to avoid complexities that are not of major importance in the problem at this stage, the treatment in this paper is not relativistic.

In the case of a dipole field, its scale length is given by (1) as follows

$$L = \frac{r}{3\sqrt{2}}\left(\frac{1+3\cos^2\phi'}{1+2\cos^2\phi'}\right)^{\frac{1}{2}}. \tag{8}$$

Here ϕ' denotes the co-latitude of the point considered and r its radial distance from the origin. Hence L is less than r by a factor that varies between 3·57 (for $\phi' = 0^\circ$) to 4·24 (for $\phi' = 90^\circ$). At 5 earth radii from the centre of the earth, in the equatorial plane, L is 7940 km. For $H = 2\cdot56\times10^{-3}$ G (the intensity of the earth's field at that distance), the radius of gyration R_p for 100 keV protons is of order 179 km, and R_e for 30 keV electrons is 1·66 km. These are much less than L.

Equation (2) may be written as follows,

$$\frac{1}{T} = \frac{1}{H}\frac{\mathrm{d}H}{\mathrm{d}t}. \tag{8'}$$

At a distance of 5 earth radii, we may assume that the maximum change of the field, taken to be 500 γ, is attained in 6 h or more, so that T is of order 10^4 s. There the period P_p for 100 keV protons is 0·256 s, and P_e for 30 keV electrons is $1\cdot48\times10^{-4}$ s. These are much less than T.

Another example is small hydromagnetic fluctuations. Consider a plane wave $h\sin(\omega t+kx)$ travelling through a region where the mean intensity is H. Equations (1) and (2) give

$$L \simeq \frac{H+h}{hk} = \left(\frac{H+h}{h}\right)\frac{\lambda_w}{2\pi}, \tag{1'}$$

$$T \simeq \frac{H+h}{h\omega} = \left(\frac{H+h}{h}\right)\frac{P_w}{2\pi}, \tag{2'}$$

where λ_w and P_w, respectively, denote the wavelength and the period of the hydromagnetic waves. The wavelength λ_w is of order 200 km at 5 earth radii (cf. Akasofu 1956). There the wave is non-dispersive. For $h = 10\gamma$ and $P_w = 20$ s, $L \simeq 4\lambda_w = 800$ km, larger than the R_p of 100 keV protons there; $T \simeq 4P_w = 80$ s, much larger than P_p. But for shock waves, in which h is comparable with H, $L \simeq \lambda_w/\pi = 64$ km, less than the R_p of 100 keV protons. Therefore, such protons do not satisfy the first condition and will be scattered by the waves. It may be noticed that the higher the energy of the particles, the more they have a tendency to be scattered. Their motion will then no longer be of the form discussed in § 3.

The drift velocity $\boldsymbol{u}_n$ perpendicular to the lines of force of the magnetic field is given (cf. Parker 1957) by

$$\boldsymbol{u}_n = c(\boldsymbol{E}\times\boldsymbol{H})/H^2 + (\tfrac{1}{2}mw_n^2c/eH^4)\,\boldsymbol{H}\times\nabla H^2/2 + (mw_s^2c/eH^4)\,\boldsymbol{H}\times[(\boldsymbol{H}\,.\,\nabla)\,\boldsymbol{H}], \tag{9}$$

where $\boldsymbol{E}$ = the intensity of the electric field.

The drift motion of the particles produces electric currents. The total current produced by the above drift motion and by the gyration is given (cf. Parker 1957) by

$$\boldsymbol{i}_n = (c/8\pi p_m)\,\boldsymbol{H}\times\{\nabla p_n + [(p_s-p_n)/p_m]\,(\boldsymbol{H}\,.\,\nabla)\,\boldsymbol{H}/8\pi + \rho\,\mathrm{d}\boldsymbol{v}_n/\mathrm{d}t\}, \tag{10}$$

where

p_s = the total pressure component parallel to $\boldsymbol{H}$, namely mnw_s^2,
p_n = the total pressure component perpendicular to $\boldsymbol{H}$, namely $\tfrac{1}{2}mnw_n^2$,
n = the number density of charged particles,
$p_m = H^2/8\pi$,
$\boldsymbol{v}_n = c(\boldsymbol{E}\times\boldsymbol{H})/H^2$,
ρ = the mass density, namely mn.

With the above notation, (9) may be rewritten thus,

$$\boldsymbol{u}_n = (c/8\pi p_m)\,(\boldsymbol{E}\times\boldsymbol{H}) + (c/8\pi nep_m)\,\boldsymbol{H}\times\{\tfrac{1}{2}(p_n/p_m)\,\nabla p_m + (p_s/p_m)\,(\boldsymbol{H}\,.\,\nabla)\,\boldsymbol{H}/8\pi\}, \tag{11}$$

(b) *Two adiabatic invariants*

When the 'guiding centre' concept is properly applicable, according to the discussion in § 3 (a), two adiabatic invariants of the motion are available to help to determine the motions of the particles (cf. Chew, Goldberger & Low 1956; Rosenbluth & Longmire 1957). They are

$$\tfrac{1}{2}mw_n^2/H = \mu, \tag{12}$$

$$\int_m^{m'} w_s\,\mathrm{d}l = J. \tag{13}$$

Here μ denotes the magnetic moment connected with the gyratory motion of a charged particle, and dl is an element of a line of force. In the earth's magnetic field the charged particles will drift slowly eastwards or westwards with rapid oscillatory motion along the lines of force between the northern and southern high latitudes, where they will be reflected. The points of reflexion are called the 'mirror points'; m and m' refer to the mirror points in the northern and southern hemisphere, respectively.

(i) Consider the motions in a dipole magnetic field. Because of the axial symmetry and the symmetry relative to the equatorial plane, equations (12) and (13) are independent of the longitude. Therefore the surface defined by (12) and (13) is made by rotating a part of a line of force around the dipole axis.

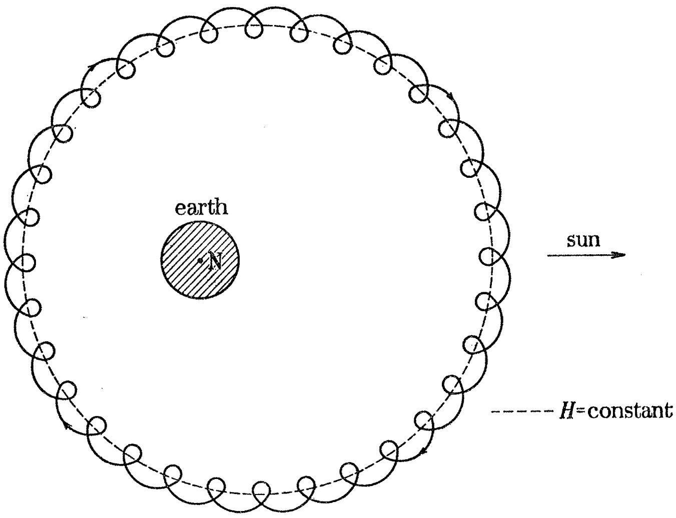

FIGURE 1. Schematic diagram illustrating the westward drift motion of a proton in the equatorial plane, in the earth's field distorted by the DCF field. The eccentricity of the particle path relative to the centre of the earth is much exaggerated. (See § 3 (*b*).)

(ii) Next consider the motion of particles in the earth's dipole field distorted by the DCF field. The earth's field is assumed to be an ideal dipole field. The simplest motion corresponds to

$$\int_{m}^{m'} w_s \, \mathrm{d}l = 0. \tag{14}$$

As w_s and dl are essentially positive, this requires

$$w_s = 0. \tag{15}$$

Hence also

$$w_n = w. \tag{15'}$$

Then, equations (12) and (13) imply that H is constant along the path of the guiding centre, so that particles go around the earth along a line given by H = constant, in the equatorial plane. Such a trajectory for a proton is schematically shown in figure 1. The field intensity is increased on the day-side hemisphere by the electric current flowing near the surface of the solar stream. Thus on that side of the earth a proton moves at a greater distance from the earth than on the night side. The mean path may not be circular, as shown, and the path may not be re-entrant or truly periodic, as shown.

The westward motion of protons shown in figure 1 is due to the gradient of the earth's field (cf. equation (9) or (11)). But for a layer of charged particles in the equatorial plane this motion does not produce any electric current, if the thickness of the layer exceeds the radius of gyration of the particles. In fact, the westward electric current due to this drift motion, which arises from the first term in the bracket in (11), namely

$$(c/8\pi e p_m)\, \boldsymbol{H} \times \{\tfrac{1}{2}(p_n/p_m)\, \nabla p_m\},$$

is cancelled by the current due to the gyration of the particles. Thus the current due to (∇p_m) does not appear in (10).

In a case not axially symmetric, the particle motions are decidedly complex. By using the relations

$$\mu = \frac{\frac{1}{2}mw_n^2}{H} = \frac{\frac{1}{2}mw^2}{H_m} = \frac{\frac{1}{2}mw^2}{H_{m'}}, \tag{16}$$

and

$$w^2 = w_s^2 + w_n^2, \tag{17}$$

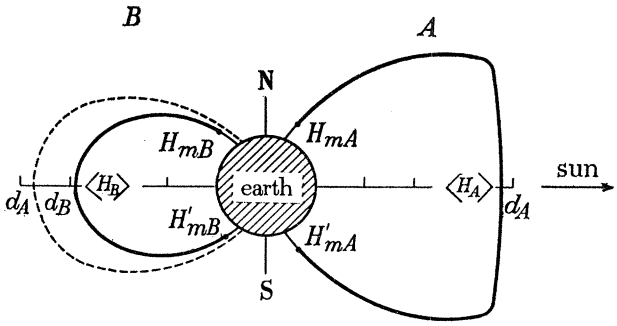

FIGURE 2. Schematic sketch of the earth's field distorted by the DCF field, in the plane through the earth's axis and the sun. The sun is taken to be in the equatorial plane. The day and night sides are indicated by A, B respectively. The thick lines indicate the cross-section of the surface J = constant. This surface crosses the earth–sun line at different distances, d_A, d_B, on the day and night sides. The broken line shows the line of force that crosses the earth–sun line on the night side, at distance d_A. (See § 3 (*b*).)

equation (13) may be rewritten as follows,

$$J = \int_m^{m'} w_s \, dl = w \int_m^{m'} \left(1 - \frac{H}{H_m}\right)^{\frac{1}{2}} dl. \tag{18}$$

Equations (16) and (17) indicate that the magnetic field intensity H_m (or $H_{m'}$) at the mirror point is a function of μ (cf. Van Allen *et al.* 1959). Also

$$H_m = w^2 H / w_n^2. \tag{19}$$

For a particle starting from the equatorial plane, we may write (19) in the form

$$H_m = w^2 \langle H \rangle / w_n^2, \tag{19'}$$

where $\langle x \rangle$ signifies the value at the point where the path crosses the equatorial plane. Therefore, for a fixed value of $\langle H \rangle$, the larger $\langle w_n \rangle$ is (or the larger the pitch angle $\langle \theta \rangle$), the smaller is H_m. The distribution of μ among a number of the particles may be denoted by $f(\mu)\, d\mu\, dV$ (dV denotes the volume element), which depends on the mechanism of capture of the particles from the solar cloud or stream. Chew *et al.* (1956) showed that the Boltzmann function f_0 is expressed by $f_0(\mu, J)$.

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When the earth's field is distorted as shown in figure 2, the value of J/w or

$$\int_{m'}^{m} (1-H/H_m)^{\frac{1}{2}}\,\mathrm{d}l$$

will depend on the longitude. The value will be less on the day side of the earth than on the night side, for integrals along lines of force that cross the equator at equal distances from the earth. This implies that a particle in its drift round the earth will approach the earth more closely on the night than on the day side.

The integral invariant (13) is strictly conserved throughout the motion only when an electric field is absent or negligibly small. It has been shown by Northrop & Teller (1960) that in the presence of an electrostatic field, or of an electric field produced by a time change of the magnetic field, $\mathrm{d}J/\mathrm{d}t$ is not zero at each instant, but that

$$\left[\frac{\mathrm{d}J}{\mathrm{d}t}\right] = \frac{1}{T}\oint \frac{\mathrm{d}s}{w_s}\frac{\mathrm{d}J}{\mathrm{d}t} = 0, \tag{20}$$

where $[x]$ signifies the average value of x during a complete cycle of rapid oscillation of the particle along a line of force between the mirror points, with period T.

If the magnetic field $\boldsymbol{H}$ is constant, and there is an electrostatic field $\boldsymbol{E}$ transverse to $\boldsymbol{H}$, due to charge separation, then

$$-\frac{1}{T}\frac{\partial J}{\partial t} = \dot{K}, \tag{21}$$

$$\dot{K} = e\boldsymbol{u}_n\,.\,\boldsymbol{E}. \tag{22}$$

If $\boldsymbol{u}_n\,.\,\boldsymbol{E} < 0$ (that is, if the directions of the vectors $\boldsymbol{u}_n$ and $\boldsymbol{E}$ are partly opposed), protons will migrate from their original surface defined by $J = \text{constant} = C_1$ to $J = C_n$, where $C_1 < C_n$.

4. The ring current and the formation of a neutral line

(a) *The ring current*

The decrease of horizontal intensity of the earth's magnetic field during the main phase of a magnetic storm has been ascribed to the generation or growth of a westward electric ring current flowing around the earth. The radiation belts involve a westward electric current flow. Thus an enhancement of the density of the radiation belts will decrease the horizontal geomagnetic component. If the state is steady $(\mathrm{d}/\mathrm{d}t = 0)$, equation (10) gives

$$\begin{aligned} \boldsymbol{i}_n &= \frac{c(p_s-p_n)}{64\pi^2 p_m^2}\{\boldsymbol{H}\times(\boldsymbol{H}\,.\,\nabla)\,\boldsymbol{H}\} + \frac{c}{8\pi p_m}\boldsymbol{H}\times\nabla p_n \\ &= \frac{cnm}{Hr_c}(w_s^2-\tfrac{1}{2}w_n^2)\,(\boldsymbol{r}_0\times\boldsymbol{H}_0) - \frac{c}{H}(\nabla p_n\times\boldsymbol{H}_0). \end{aligned} \tag{23}$$

Here r_c $(=|r_c\boldsymbol{r}_0|)$ denotes the length of the radius of curvature, at the point P considered, of the line of force along which the particle moves; $\boldsymbol{r}_0$ denotes the unit vector of the radius of curvature. Here we have used the vector relations

$$(\boldsymbol{H}\,.\,\nabla)\,\boldsymbol{H} = -H^2\boldsymbol{r}_0/r_c, \tag{24}$$

and

$$\boldsymbol{H} = H\boldsymbol{H}_0.$$

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In the dipole field
$$1/r_c = \frac{3(1+\sin^2\phi)}{d\cos\phi(1+3\sin^2\phi)^{\frac{3}{2}}}, \tag{25}$$
$$H = H_M = \langle H\rangle\,(1+3\sin^2\phi)^{\frac{1}{2}}/\cos^6\phi, \tag{26}$$
$$H = 0{\cdot}32/(d/a)^3, \tag{27}$$
where

ϕ = latitude,

d = the distance from the earth's centre to the point $\langle P\rangle$ where the line of force crosses the equatorial plane,

a = the radius of the earth.

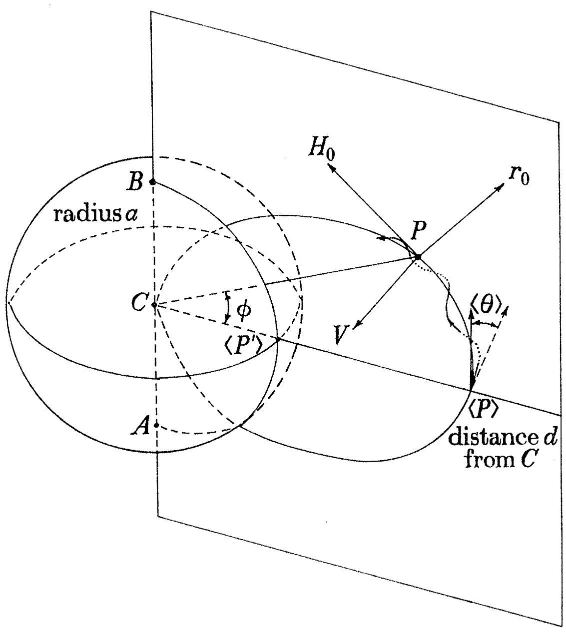

FIGURE 3. To illustrate the parameters that determine the trajectory of a charged particle in the earth's field. B (for boreal) and A (for austral) denote the geomagnetic north and south poles, respectively. (See § 4 (*a*).)

The first term in the bracket of (23), namely
$$\boldsymbol{i}_c = \frac{cmn}{Hr_c} w_s^2(\boldsymbol{r}_0 \times \boldsymbol{H}_0) = neV, \tag{27'}$$
is the current due to the centrifugal force exerted on the particles moving along the curved lines of force (denoted by $\boldsymbol{i}_c$). The protons drift westward with speed V_p and the electrons drift eastward with speed V_e. (Note that V is a function of m.) Both contribute to the westward ring current. It may be noticed that the centrifugal force exerted on the particles is balanced by the Lorentz force which is the vector product of the westward current and the earth's field, namely $\boldsymbol{i}_c \times \boldsymbol{H}$ (see figure 3).

(*b*) *The formation of a neutral line*

The above relations are strictly valid only for infinitely small current density. Actually the westward current must appreciably modify the normal field of the earth. This change affects the motion of the charged particles and so modifies the ring current itself. Consider, for illustration, the combination of a uniform magnetic field $\boldsymbol{H}$ with the field of an infinitely long straight current, perpendicular to $\boldsymbol{H}$, flowing with uniform density $\boldsymbol{i}$ in a cylindrical volume with circular cross-section. Typical field distributions have been calculated by McDonald (1954), and are shown in figure 4 for different ratios i/H.

Figure 4*a* shows the lines of force when the volume current is zero. Figure 4*b* shows the lines of force when the current is weak. When the current density is sufficiently increased, a neutral line appears in the magnetic field, first on the surface of the current distribution (figure 4*c*). For greater current densities there are two neutral lines, one inside and one outside the cylinder, as in figure 4*d*. The inner neutral line intersects the plane of the diagram in a point marked O; there the lines of force shrink to a point. The outer neutral line intersects the plane of the diagram in a point marked X, to indicate that the line of force through it is double; this line of force has two branches that may extend to infinity.

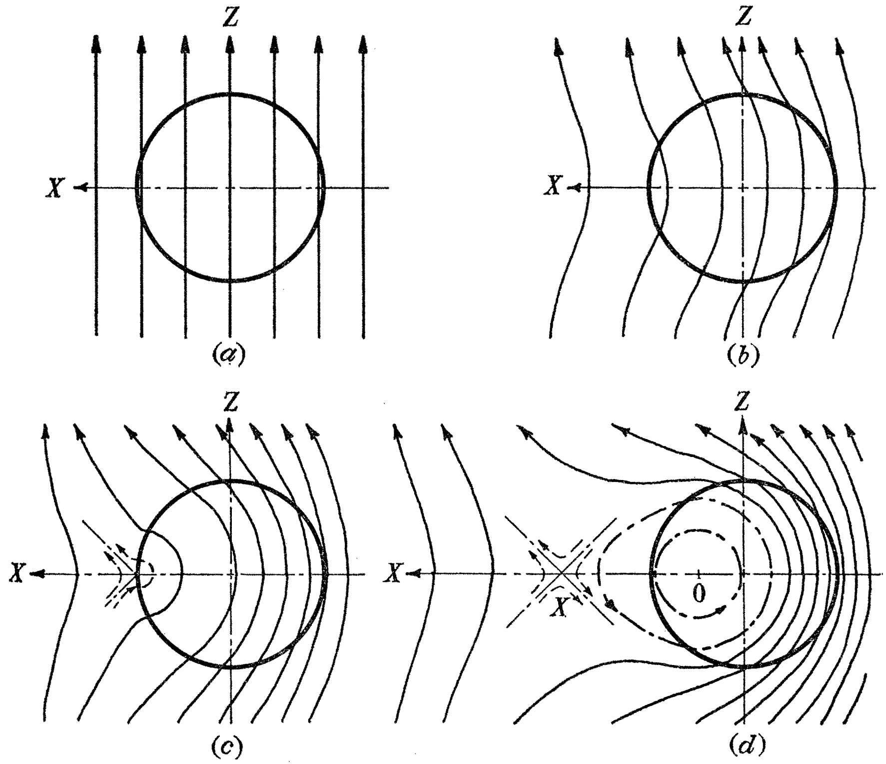

FIGURE 4. To illustrate the distortion of a uniform field by a uniform volume electric current (flowing from the paper) along and throughout a cylinder whose axis is normal to the field: for values of the volume current that increase from zero in case (*a*) to a large value in case (*d*). The X- and O-type neutral lines are also shown. (After McDonald 1954.) (See § 4 (*b*).)

Figure 4 is drawn for a uniform current density. In the case of an actual ring current, in which the distribution of the current density may be non-uniform (and variable), the distortion may be much more than that shown in figure 4. The cylindrical current always distorts the original straight field lines so that the radius of curvature r_c is decreased. Thus the actual ring current must increase the current density, as equation (23) shows. Another important effect is a reduction of the intensity of the field to the left of the median plane of the cylinder (figure 4). Both effects modify the original particle motions so as to increase the current density.

Figure 5 is a schematic sketch of the magnetic field configuration in a meridian half-plane, when a toroidal ring current encircles the earth, strong enough to produce neutral lines in the field. These lines, which are circles with the same centre as that of the earth, intersect the plane of the diagram in the neutral points O and X (as in figure 4). The centre line of the ring current in figure 5 has a radius 5·5 times that of the earth.

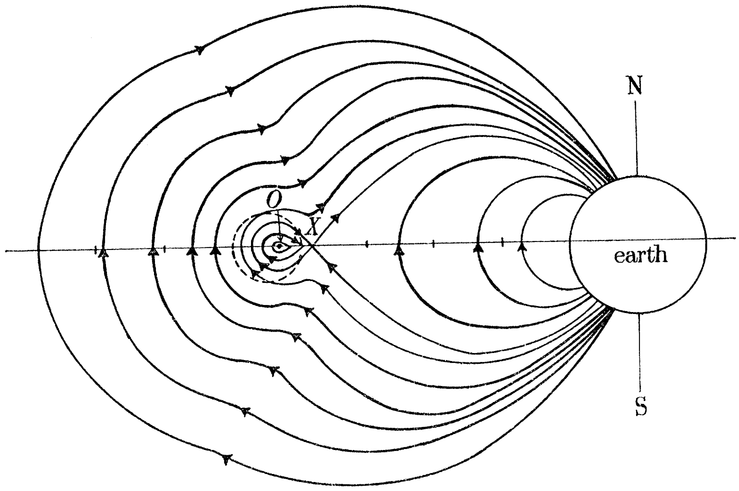

FIGURE 5. Schematic diagram of the magnetic field configuration in a meridian half-plane, when a toroidal ring current is flowing round the earth. The boundary of the cross-section of the toroid is indicated by the broken line. The X- and O-type neutral lines are also shown. (See § 4 (*b*).)

(*c*) *The field intensity distribution*

To a first approximation we here regard the ring current as a toroidal ring current of circular cross-section. Let a, q and b denote the radii of the earth, the centre-line of the ring and the boundary of the cross-section of the ring. The magnetic field H_R produced by the ring, in the equatorial plane, outside the ring, may approximately be given (cf. Stratton 1941, p. 263) by

$$H_R = 2I\frac{1}{c(q+d)}\left(K+\frac{q^2-d^2}{(q-d)^2}E\right), \quad 0<d<(q-b). \tag{28}$$

Here I denotes the total current intensity in the ring; $K(k)$ and $E(k)$ are complete elliptic integrals of the first and second kinds, respectively; d denotes the distance from the centre of the earth, and

$$k^2 = 4qd/(q+d)^2. \tag{29}$$

The field intensity at C the centre of the earth is

$$H = 2\pi I/qc \quad (d=0). \tag{30}$$

Within the volume of the ring the field distribution may approximately change linearly along any radius. At the centre of the cross-section of the ring the field produced by the ring is almost zero.

As discussed in § 2, the field DCF, produced by the currents flowing near the surface of the hollow in the solar stream, is very roughly equal to that of a dipole with the same magnetic moment as the earth. The dipole is on the line SCS' from S the sun to C the centre of the earth and beyond. Its distance from C is $2A$, towards S; here A denotes the distance from C to the vertex of the hollow. The intensity is H_{CF}. The intensity of the compound field is $\boldsymbol{H}$, the vector sum of the intensities $\boldsymbol{H}_M$, $\boldsymbol{H}_{CF}$, and $\boldsymbol{H}_R$. Neglecting DP, the field of the polar currents, $\boldsymbol{H}$ will be an approximation to the field existing round the earth during a storm. At points along the line SCS' all these fields are parallel or antiparallel.

Some illustrative calculations of H have been made, with the choice of what seem to be reasonable values of A/a, q/a and b/a, namely

$$A/a = 6, \quad q/a = 5{\cdot}5, \quad b/a = 0{\cdot}5.$$

Four values of the intensity H_{R0} of the field of the ring current at ground level at the earth's equator are considered,

$$H_{R0} = 50,\ 70,\ 100,\ 500\gamma.$$

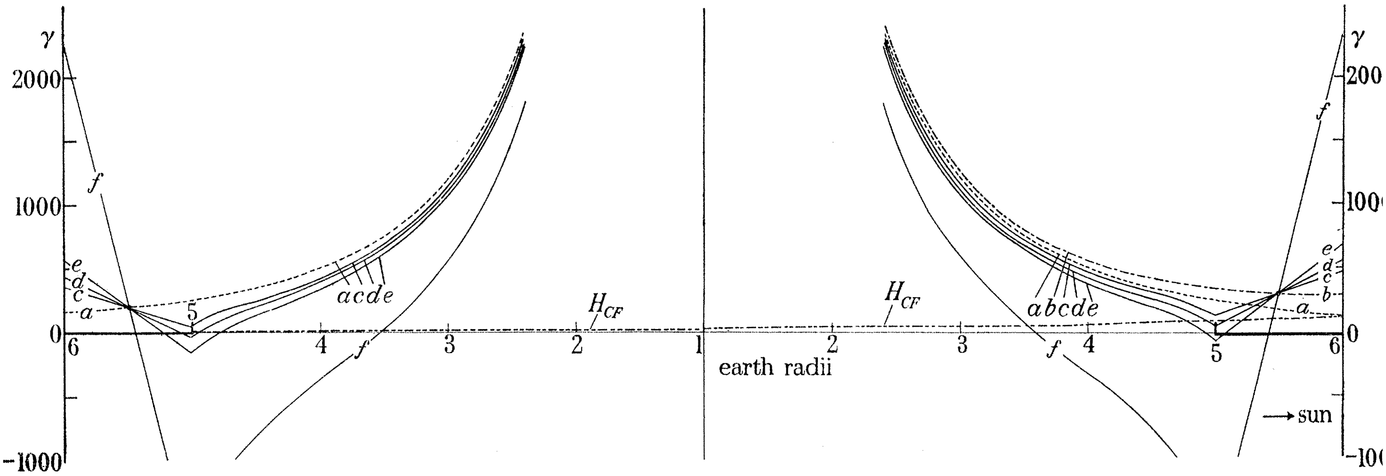

FIGURE 6. Schematic diagram of the magnetic field intensity along the sun–earth line for various values of H_{R0}. Left, on the night side, right, on the day side, from the earth's surface to a distance of 6 earth radii. H_M, H_{CF} and (H_M+H_{CF}) are also shown. (See § 4 (*c*).) (*a*) H_M; (*b*) H_M+H_{CF}; (*c*) $H_M+H_{CF}+H_R$ for $H_{R0} = 50\gamma$; (*d*) $H_M+H_{CF}+H_R$ for $H_{R0} = 70\gamma$; (*e*) $H_M+H_{CF}+H_R$ for $H_{R0} = 100\gamma$; (*f*) $H_M+H_{CF}+H_R$ for $H_{R0} = 500\gamma$.

These cover the whole likely range of values of H_{R0}, which of course depends on q/a and the current strength along the ring. In figure 6 the graphs of H for these various cases are shown, on the right for points along CS, on the day side of the earth, and on the left, for points along CS', on the night side. (The earth itself is omitted from the diagram.) The graph of H_{CF} is shown. It must have a small discontinuity in the centre of the diagram, owing to the omission of the earth's diameter. On the right the addition of H_{CF} appreciably modifies the curves of H_M and H_M+H_R. On the left H_{CF} is too small to change these curves appreciably. On the right, at 6 earth radii, at the position of the vertex of the hollow, H_{CF} and H_M are equal. Despite the small intensity of the field DCF, this field produces an important difference between the values of H on the two sides of the earth. Figure 6 shows neutral points on the day side only for rather large values of H_{R0}, 100 and 500γ. These values would correspond, with the field of the currents induced in the earth, to about 160 and 800γ decrease during the main phase of the storm (owing to the additional field of the currents induced in the earth). The difference between the curves to the right and left should be specially noticed in connexion with the appearance of neutral lines in the field.

The deviation of the earth's external field from the expected dipole field has recently been observed by the U.S.S.R. cosmic rocket Mechta (2 January 1959) and by the U.S. satellite Explorer VI (10, 24 August; 1 September 1959; Sonett, Smith, Judge & Coleman 1960). The field distribution for $H_{R0} = 50\gamma$ seems to agree qualitatively with their observations.

For $H_{R0} = 70\gamma$ the direction of the composite field is reversed near the ring on the night-side meridian, but not on the day-side meridian. Figure 7 shows the geometry of the X- and O-type neutral lines in the equatorial plane in this case. They join at a point on the surface of the ring current. Figure 7 also shows the projection of the X-type neutral line on the earth's surface, following the dipole lines of force that intersect the X line. This projection is the locus along which particles from the neutral line will enter the earth's atmosphere. Their entry will be made visible by a long thin auroral arc in the east–west direction.

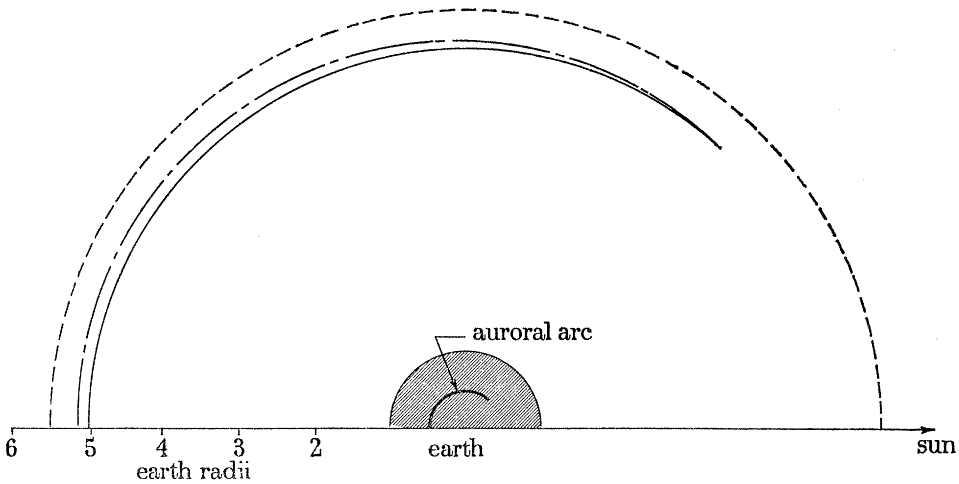

FIGURE 7. Geometry of the X- and O-type neutral lines in the equatorial plane for $H_{R0} = 70\gamma$. The projection of the X-type neutral line on the earth's surface is also shown. (See § 4 (c).). ——, X-type neutral line; — — —, O-type neutral line; - - -, centre of the cross-section of the ring current.

For $H_{R0} = 100\gamma$ the region of negative H, and the neutral lines, will completely surround the earth. For $H_{R0} = 500\gamma$ the negative region is much enlarged, and one of the neutral lines (X-type) draws closer to the earth. So far we have assumed that the ring current is concentric around the earth. However, the ring may not be concentric. An exact treatment must follow the discussion given in § 3 (b). As was shown there, by using the two adiabatic invariants, it is expected that the ring will be nearest to the earth on the night-side hemisphere. This is again discussed in § 6 (b).

As indicated later (§ 7 (b) and figure 14) the ring current may often be less simple than has been supposed above. It may have different parts, located between 4·5 and 7·5 earth radii. It may be partly eastward, deepening the decrease of H (see our paper 1961 c).

5. Motions of charged particles close to a neutral line

(a) *Motions of charged particles in the equatorial plane*

In the foregoing discussion the conditions assumed in § 3 (*a*) were supposed to be satisfied. The first two may be written thus:

$$R/L \ll 1,$$
$$P/T \ll 1.$$

Here L and T denote the scale length and scale time defined by (1) and (2). However, these two conditions are not satisfied near a neutral point or line. We take rectangular Cartesian co-ordinates such that the neutral line coincides with the y axis (or $x = z = 0$) and so that all the field lines are contained in the xz plane. Further we direct the x and z axes in such a way that the magnetic potential Ψ is expressed by

$$\Psi = \Psi_0 - fxz. \tag{31}$$

Then the $\boldsymbol{H}$ field components (X, Y, Z) are

$$X = fz, \quad Y = 0, \quad Z = fx. \tag{32}$$

Here

$$f = \partial X/\partial z = \partial Z/\partial x. \tag{33}$$

Chapman (1960) has shown that in this case the scale length defined by (1) is

$$L = r/\surd 2, \tag{34}$$

where

$$r = (x^2 + y^2)^{\frac{1}{2}}. \tag{35}$$

Hence near the neutral line L becomes small, but R and T become large, so that $R/L \gg 1$ and $P/T \gg 1$. Therefore the guiding centre concept can no longer be applied there, and the true paths of the particles must be considered. This has been done by Åström (1956) and Parker (1957). Here we present the analysis in a simple form.

By the use of (32), the equations of motion of charged particles (velocity components (u, v, w)) near the neutral line are thus obtained. Elsewhere in this paper w signifies the total speed of a particle; here exceptionally it is used for the z component of the velocity.

$$\left.\begin{aligned} m\dot{u} &= m\ddot{x} = (e/c)\,vZ = (fe/c)\,xv, \\ m\dot{v} &= m\ddot{y} = (e/c)\,(wX - uZ) = (fe/c)\,(zw - xu), \\ m\dot{w} &= m\ddot{z} = (e/c)\,(-vX) = -(fe/c)\,zv. \end{aligned}\right\} \tag{36}$$

We consider the motion of charges in the plane $z = 0$. Then the above equations are

$$\left.\begin{aligned} \dot{u} &= \ddot{x} = (ef/mc)\,xv = Axv, \\ \dot{v} &= \ddot{y} = (ef/mc)\,(-xu) = -Axu, \\ \dot{w} &= \ddot{z} = 0, \end{aligned}\right\} \tag{37}$$

where

$$A = ef/mc. \tag{38}$$

We note that if we simultaneously change the sign of the charge, e, and the direction of the y axis, the form of the equation remains the same. This indicates that positive and negative particles have opposite directions of motion along the y axis. Note also that for particles of different mass the path is the same except in scale. The first integral is

$$u^2 + v^2 = \text{constant} = w_n^2. \tag{39}$$

From (37) and $\dot{v} = u\,\mathrm{d}v/\mathrm{d}x$, v is obtained as

$$v = v_0 - \tfrac{1}{2}Ax^2, \tag{40}$$

where $v = v_0$ at $x = 0$ (that is, v_0 is the velocity component along the y axis when the particle crosses the neutral line). Similarly for u

$$u^2 = u_0^2 + Av_0x^2 - \tfrac{1}{4}A^2x^4. \tag{41}$$

Here we put $u = u_0$ at $x = 0$ (that is, u_0 is the velocity component along the x axis when the particle crosses the neutral line). We may write (41) in the form

$$u^2 = \tfrac{1}{4}A^2(X_1^2 - x^2)\,(X_2^2 + x^2), \tag{42}$$

so that $u = 0$ at $\pm X_1$. Also

$$\int \mathrm{d}t = \frac{2}{A}\int \frac{\mathrm{d}x}{\{(X_1^2-x^2)\,(X_2^2+x^2)\}^{\frac{1}{2}}}, \tag{43}$$

where
$$X_1^2 = \frac{2v_0}{A}\left(1+\frac{w_n}{v_0}\right), \tag{44}$$

$$X_2^2 = -\frac{2v_0}{A}\left(1-\frac{w_n}{v_0}\right). \tag{45}$$

Similarly
$$\int \mathrm{d}y = \frac{2}{A}\int \frac{(v_0-\frac{1}{2}Ax^2)\,\mathrm{d}x}{\{(X_1^2-x^2)\,(X_2^2+x^2)\}^{\frac{1}{2}}}. \tag{46}$$

The travel time τ between $x = 0$ and $x = X_1$ is then

$$\tau = \frac{2}{A}\int_0^{X_1} \frac{\mathrm{d}x}{\{(X_1^2-x^2)\,(X_2^2+x^2)\}^{\frac{1}{2}}}. \tag{47}$$

This can be expressed as
$$\tau = \frac{1}{\surd(Aw_n)}E(k), \tag{48}$$

where $E(k)$ is the complete elliptic integral of the first kind, and

$$k^2 = \tfrac{1}{2}(1+v_0/w_n). \tag{49}$$

Similarly the displacement η along the y axis during the above time interval is

$$\eta = (mw_nc/fe)^{\frac{1}{2}}\{E(k)-2K(k)\}, \tag{50}$$

where $K(k)$ is the complete elliptic integral of the second kind. Therefore the mean drift velocity of the particle $\overline{V}$ along the neutral line is

$$\overline{V} = \frac{\eta}{\tau} = w_n\left\{1-\frac{2K(k)}{E(k)}\right\}. \tag{51}$$

For $k > 0{\cdot}909$ the particle drift is in one direction, and for $k < 0{\cdot}909$ it is in the opposite direction. For positive charges the former direction is positive, for electrons it is negative.

It is seen from (49) that k^2 is a function of (v_0/w_n) or $\cos\chi$, where χ denotes the angle between the neutral line and the velocity vector at a point where the path intersects the neutral line. By definition

$$-w_n \leqslant v_0 \leqslant w_n,$$

so that
$$0 \leqslant k^2 \leqslant 1. \tag{52}$$

For particles moving from a point on the neutral line with $v_0 > 0$ (that is, in the positive y direction, so that $k^2 > \frac{1}{2}$), the path is looped. For positively charged particles moving from the y axis in the negative direction ($v_0 < 0$), so that $k^2 < \frac{1}{2}$, the path is of simple wave form, and always in the negative y direction. This is the kind of motion with which we are concerned in the later part of this paper.

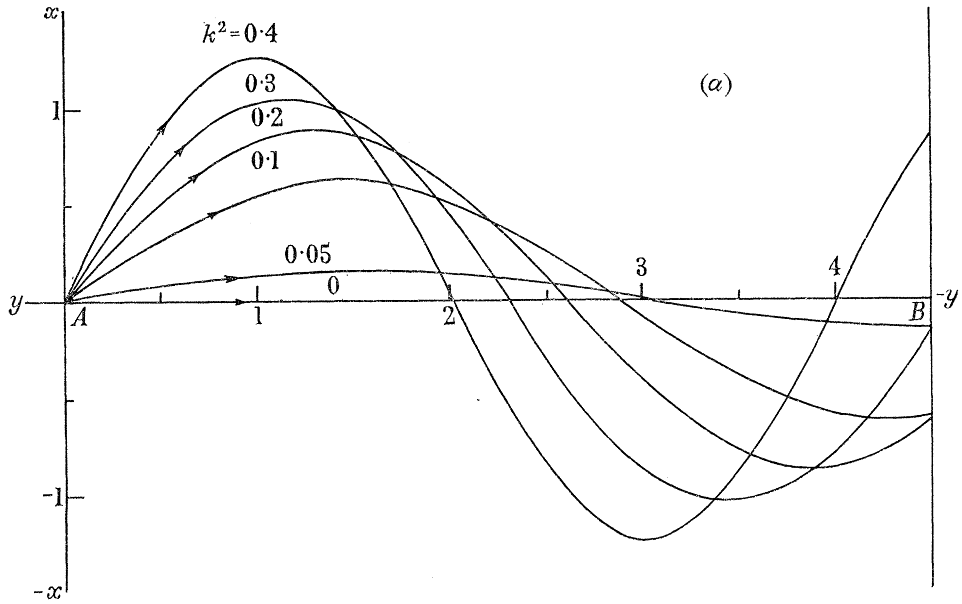

FIGURE 8. (a) Paths of protons near the X-type neutral line for various values of k^2. The scale unit of length is $(mw_{np}c/ef)^{\frac{1}{2}}$. The paths lie in the equatorial plane; AB is part of the neutral line along the y direction; the curvature of this line is too small to be shown in this diagram. (See § 5 (a).)

The paths of protons for $k^2 = 0$, 0·05, 0·1, 0·2, 0·3 and 0·4 are calculated and are shown in figure 8 (a). The scale unit of length is $(m_p w_{np} c/ef)^{\frac{1}{2}}$. The electron drift velocity $\overline{V}_e$ along the neutral line is

$$\overline{V}_e = w_{ne}\left\{\frac{2K(k)}{E(k)} - 1\right\}, \tag{53}$$

and the scale unit of length is $(m_e w_{ne} c/fe)^{\frac{1}{2}}$.

The maximum width of the region in which the motion along the neutral line is unidirectional corresponds to $k^2 = \frac{1}{2}$, in which case

$$v_0 = 0,$$

and $$X_1 = (2w_n/A)^{\frac{1}{2}} = (2mcw_n/ef)^{\frac{1}{2}}. \tag{54}$$

Thus for 130 keV protons $$X_{1p} = 3{\cdot}2\times 10^2/\surd f, \tag{55}$$

and for 30 keV electrons $$X_{1e} = 3{\cdot}3\times 10/\surd f. \tag{56}$$

Therefore the width for the electrons is about $\frac{1}{10}$ of that for the protons.

From the above considerations, protons can drift in both the positive and the negative directions. Thus the direction of the net drift and hence of the electric current j depends on the initial condition—the distribution function of k or k^2. However, Parker (1957, p. 933) infers that a steady state will eventually be established. Within the width $2X_1$ the current j is related to the gradient of the field in the following way.

$$j_y = \frac{-c}{4\pi}\left(\frac{\partial Z}{\partial x}\right)_y. \tag{57}$$

(b) Growing electric current along the neutral line

In a non-conducting gas the lines of force that meet at a point on an X-type neutral line cross each other orthogonally. In § 5 (a) it has been shown that electric current will flow along the strip adjacent to an X-type line within the width $2X_1$. The angle between the two lines of force which cross on the neutral line is thereby changed (because $\partial Z/\partial x$ changes). The gradient of the field near the neutral line increases, according to (57). Figure 8 (b) shows schematically the distribution of the magnetic field intensity around the neutral line along the x axis. As has been shown in (55) and (56), X_{1e} for 30 keV electrons is about one-tenth of X_{1p} for 130 keV protons. Electrons between X_{1p} and X_{1e} drift along the y axis with various trochoidal orbits, according to the first term in the bracket in (11). These electrons cannot cross the neutral line, and their guiding centres move parallel to the y axis (the neutral line). Several such orbits are discussed by Åström (1956).

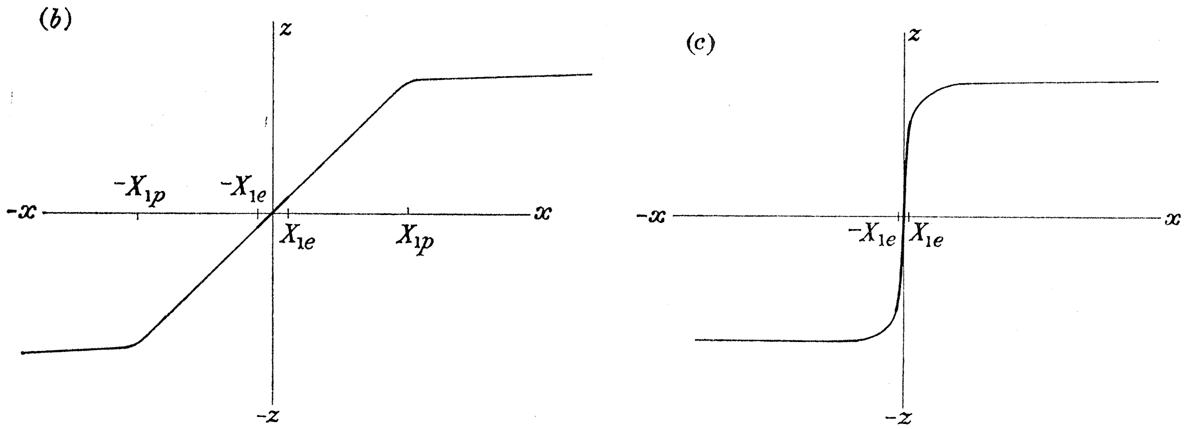

FIGURE 8 (b) Schematic distribution of the magnetic field intensity near the X-type neutral line. The protons (energy 130 keV) drift in the negative y direction (from the paper) with oscillatory motion of amplitude X_{1p}. The electrons (30 keV) drift in the positive y direction (into the paper) with the amplitude X_{1e}. (See §§ 5 (a, b), 7 (d).)

8 (c) Schematic distribution of the magnetic field intensity near the X-type neutral line, when an electric field appears along the positive y direction. Note the increased gradient of the field near the neutral line. (See §§ 5 (b) and 8.)

Dungey (1953) showed that this state will not be steady, and that the electric current will grow spontaneously. He inferred that the gradient ($\partial Z/\partial x$) in the narrow strip around the X-type neutral line will tend to become infinite. It may be, however, that the region between $\pm X_{1p}$ has a structure similar to that of the front layer of hydromagnetic shock waves. In this transition layer various physical quantities, such as the pressure gradient ∇p_n and the viscosity, will become important. They tend to reduce the high field gradient there. Petschek (1958) concluded that the thickness of the transition layer is of the same order as the radius of gyration of the protons. Therefore we may infer that X_{1p} is of order R_p just outside the narrow strip (see figure 8 (b)); that is,

$$X_{1p} \simeq (cm_p w_p/eZ) = R_p. \tag{58}$$

Next we consider the Cartesian co-ordinate system used in this section in relation to the earth's field. We take the z axis northward, antiparallel to the earth's dipole axis. In the equatorial plane we direct the positive x axis toward the earth's centre. Then the positive y axis is westward, and the electric current j_y in (57) is eastward. The field intensity Z becomes H. Thus (57) may be rewritten as follows:

$$j_y = -\frac{c}{4\pi}\left(\frac{\partial H}{\partial x}\right)_y. \tag{57'}$$

We may also take Z, in (58), to have the value of the intensity H_M of the original earth's dipole field there, namely

$$Z = H_M. \tag{59}$$

Thus (58) is

$$X_{1p} \simeq (cm_p w_p/eH_M). \tag{58'}$$

At this point we introduce an assumption that is fundamental for the later developments in this paper. It is that an electric field may arise along the direction of the neutral line, that accelerates the spontaneous growth of the current along the line. Auroral and geomagnetic observations suggest that this electric field appears sporadically (see § 9). This electric field must be eastward (in the negative y direction in figures 8 (a), (b) and (c)). It will accelerate the particles that lie near the neutral line. The electrons will be the more accelerated in the westward direction (because of their smaller mass) opposite to that of the field. They move continually westward within the strip of width $2X_{1e}$.

Therefore the electric current may be expected to increase substantially within the width $2X_{1e}$. Thus the gradient of the field is increased according to (57′), and X_{1e} is reduced according to (56). Of course, even in this case, the field gradient will not be infinite. We suspect that X_{1e} becomes comparable with the radius of gyration of the electrons $(cm_e w_e/eH_M)$ (see figure 8 (c)).

Further the proposed electric field will cause the particles in the positive and negative x domains to draw closer to the neutral line. Outside the strip of width $2X_{1e}$, in which the particles gyrate regularly around the lines of force (which there lie along the z direction), this drift is given by the first term of (9), namely

$$\boldsymbol{v}_n = c\boldsymbol{E}\times\boldsymbol{H}/H^2.$$

Inside this strip there is no complete gyration, and the drift will be less than $c\boldsymbol{E}\times\boldsymbol{H}/H^2$. By vector multiplication by $\boldsymbol{H}$ the above equation may be transformed to

$$\boldsymbol{E}+\boldsymbol{v}_n\times\boldsymbol{H}/c = 0$$

(cf. Cowling 1956, p. 542). This indicates that a magnetic field is frozen in the plasma. Therefore there is a tendency for the particles, as they draw closer to the neutral line, to carry the lines of force with them from both sides. The concentration of particles near the neutral line is associated with an increase in the magnetic gradient $(\partial H/\partial x)$. Thus the electric field has three effects, the concentration of the particles, the steepening of the magnetic field gradient, and the acceleration of particles along the neutral line. All these effects contribute to increase the linear velocity of the particles along the neutral line.

To explain the auroral particles, however, the above drift velocity $\overline{V}$ $(=\eta/\tau)$ must be converted into a velocity component along the lines of force. Just above and below the equatorial plane, where the first invariant law is applicable, equation (19′) gives

$$H_m = Hw^2/w_n^2.$$

Taking $H_m = 0{\cdot}6$ G and $H = 5\times10^{-4}$ G (50γ), we infer that only the particles for which the ratio (w/w_n) exceeds 34·6 can reach the auroral zone. This indicates that only the particles with initial velocity nearly parallel to the line of force can penetrate into the auroral level.

When the conditions in § 3 (*a*) are satisfied, the first invariant law

$$\tfrac{1}{2}mw_n^2/H = \mu = \text{constant}$$

shows how the velocity components w_s and w_n are changed in a non-uniform field. The conditions are satisfied even for hydromagnetic waves of amplitude 10γ and period 20 s, especially as regards the electrons, because of their small radius and short period of gyration. Indeed for electrons the conditions are satisfied for hydromagnetic waves of period from 0·1 to 1 s if the amplitude does not exceed about 50γ at 5 earth radii from the earth's centre. This means that the increase of w_s can be attained only when the particles enter a region of weak field intensity, if the electric field along the lines of force is zero.

However, around the neutral line, within the width $2X_1$, the conditions of invariance no longer apply. The conversion of the velocity component w_n into w_s (or w_s into w_n) can, therefore, occur most easily within the width $2X_1$. The particle–particle collisions seem almost negligible, because the collision interval ($T^{\frac{3}{2}}/15n_e$; cf. Cowling 1956, p. 538) is more than 450 s, taking $T = 7734$ °K ($= 1$ eV) and $n_e = 10^2/\text{cm}^3$. Magnetic fluctuations seem to play an important role in this process. In figure 8 (*a*), any small fluctuating X or Y field component will produce an upward or downward force, and allow the particles to escape from the xy plane (the equatorial plane).

Åström (1956) showed that except for $0{\cdot}86 < k < 0{\cdot}91$ the particle motions we have discussed are unstable in the sense that any small deviation from the equatorial plane will increase. Thus all the motions within the width $2X_1$ are unstable. The magnetic fluctuations discussed above can easily generate much instability.

Furthermore, just above and below the equatorial plane ($z = 0$) there is the X component of the magnetic field (see equation (32)). This field continuously deflects the particles upwards or downwards. The equation of motion for the particles when just above or below the equatorial plane is (see equation (36))

$$m\dot{w} = -(fe/c)\,zv, \tag{60}$$

or

$$\ddot{z} = \tfrac{1}{2}\,\mathrm{d}\dot{z}^2/\mathrm{d}z = -(fe/mc)\,zv. \tag{60'}$$

Integrating (60′) we have

$$\dot{z}^2 = \dot{z}_0^2 - (fe/mc)\,vz^2, \tag{61}$$

where $\dot{z}_0$ ($= w_0$) denotes the initial velocity along the z axis. We are interested in the particles drifting along the neutral line with simple wave motion (namely, particles with $k < \frac{1}{2}$). For such protons, $v < 0$; thus the right-hand side of (61) is always positive. The protons, once deflected from the equatorial plane, will continue to move upward or downward from the equatorial plane. Likewise the electrons also continue to move upward or downward (because $v > 0$ and $e < 0$).

We may also expect that the intense constriction of the electrons around the neutral line produces a strong electrostatic field. This is rapidly diminished by a differential motion of the background ionization along the lines of force and also by the motion of the constricted electrons themselves along the lines of force. These electrons will gain in velocity component along the lines of force.

From the above discussion we may conclude that the region around an X-type neutral line of width $2X_1$ is the only region where the particles in the equatorial plane can be accelerated and where their velocity components w_s and w_n are interconvertible.

In the following sections we develop the foregoing arguments in relation to auroral morphology. It seems that the major auroral phenomena can be explained as the direct or indirect result of complicated electromagnetic changes occurring in or near the equatorial plane at a distance of several earth radii from the earth's centre. These changes produce associated complicated effects in the auroral atmosphere. Our hypotheses may be contrasted with Störmer's. He ascribed the auroral phenomena to solar particles coming directly from the sun, under the influence of the dipole field alone, or of this and also of the ring current. He made no mention of neutral lines, which are fundamental in our theory.

6. The auroral zones

(a) *The auroral zones*

The first great compilation of all available auroral data was made by Fritz (1873). From it he computed, for a number of stations, the average relative frequency of nights of auroral visibility, expressed as M nights per year. Then he drew a chart (1874) showing lines of equal M. These lines he called 'isochasms'. His chart referred only to the northern hemisphere. His work was revised by Vestine (1944), who used further data accumulated up to 1942. Vestine & Snyder (1945) later made a similar map for the southern hemisphere (this has been revised recently by Bond & Jacka (1960), who used the data obtained during the I.G.Y.).

The geographical distribution of the aurora thus obtained, however, gives no indication of the space-time morphology of an individual auroral display. Davis & Kimball (1960), using data from a closely spaced array of all-sky cameras that covered the major part of the Alaskan sky, showed that there, on the average, auroral arcs occur most often between geomagnetic latitudes 66 and 67°. They also found that new arcs most often originate between geomagnetic latitudes 66 and 69°, whence they move southward or northward.

We interpret these two facts as indicating that the neutral lines are most frequently formed in the equatorial region approximately at a distance d given by

$$d = 1/\sin^2\phi' = 6{\cdot}29 \text{ earth radii}, \tag{62}$$

where ϕ' denotes the auroral co-latitude. This is taken to be 23° 30′. This also suggests that at such times the main part of the ring current most often lies just beyond this distance from the centre of the earth. The major injection of solar particles, whose precise mechanism is not known, may occur there.

To explain this, Störmer (1955, p. 294) had to assume that the energy of the protons is close to 4·8 MeV. Even if the requirement regarding the proton energy were rendered less unlikely by the distortion of the earth's field (Störmer, pp. 340–46), it is altogether impossible to expect that the field can guide electrons of order from 30 to 100 keV to that region. (Even electrons of energy as high as 0·3 GeV can enter only in geomagnetic latitudes higher than 77°.) As recent researches have shown (cf. Meredith, Davis, Heppner & Berg 1958; McIlwain 1959; Winckler 1960), electrons of energy 30 to 100 keV play an important role in auroral physics. It is also against present evidence to assume that the phenomena depend on protons of well-defined energy 4·8 MeV.

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(b) *The daily variation of the auroral zones*

(i) *Night-time appearance*

The auroral zone defined by Fritz and others is of course derived from night-time observations. Radar can detect day-time auroras. It is thus found that auroras may extend into the hours of daylight and begin before sunset. But this seems to occur rarely. As far as the auroral zones are concerned, one of the most important auroral features is their occurrence chiefly at night. This can be explained by the distortion of the earth's field by the DCF field, produced by the electric current at the surface of the solar stream.

The 'dipole' field is 'compressed' and essentially doubled at the vertex of the inner surface of the stream. But it is much less affected on the night side of the earth. Therefore, as is suggested in § 4 (*c*) and in figure 7, the ring current may often be able to reverse the undisturbed earth's field on the night side, when it cannot reverse it on the day side until the ring current is strongly enhanced. Therefore we expect that during weak and medium disturbances the neutral line or lines are usually formed only on the night side.

The argument in § 6 (*a*) suggests that on the average the radius q of the ring is larger than 6·3 earth radii. Taking $(q/a) = 6{\cdot}5$, $(b/a) = 0{\cdot}5$ and $(A/a) = 6{\cdot}7$, it is found that the neutral line appears at about 6 earth radii if H_{R0}, the surface intensity of the ring at the equator, exceeds 32γ. However, if there is a more or less permanent ring current with intensity 50γ there, as is suggested by Sonett *et al.* (1960), an addition of $H_{RO} = 21\gamma$ will suffice to produce a neutral line on the night-side hemisphere. This seems quite a reasonable amount.

Thus the formation of the neutral line depends not only on the intensity of the pre-storm ring current, but also on the intensity of H_{CF} and on the position of the ring. This is further discussed in § 6 (*c*).

It is also interesting to note that when the intensity of the ring current is enhanced the neutral line extends further towards or on to the day side of the earth. In fact, auroral displays are seen in the evening and dawn twilight at times of large magnetic disturbance.

(ii) *Regular daily variation of the auroral zones*

Another important feature of the auroras is the regular daily progression towards the south before midnight and the recession towards the north after midnight. This has been discussed by Heppner (1954), Rees & Reid (1959) and Davis & Kimball (1960). This may also be explained by the distortion of the earth's field, as was first pointed out by Rees & Reid (1959), and Reid & Rees (1960). However, as was indicated in § 3 (*b*), the detailed proof must await rather laborious numerical calculation of the adiabatic invariants $\int_m^{m'} w_s \, \mathrm{d}l = J$ for such a distorted field.

Vestine & Sibley (1959) calculated this integral for the actual earth's field, by taking $H_m = 0{\cdot}45$ G and by using 48 terms of the spherical harmonic expansion of the geomagnetic field. They drew isolines of the integral $\int_m^{m'} (1 - H/H_m)^{\frac{1}{2}} \, \mathrm{d}l$; these are here reproduced as figure 9. The close agreement with the isochasms drawn by Fritz and Vestine suggests that the auroral particles may be treated in the way discussed in § 3. The instantaneous position of the aurora in the polar region will be obtainable in essentially the same way, when we can properly take into account the distortion of the (M + DR) field by DCF.

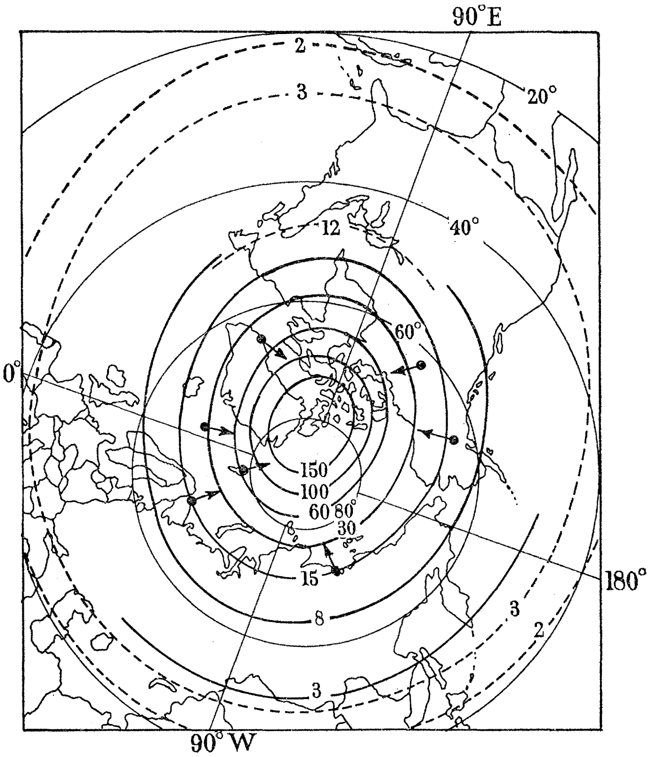

FIGURE 9. Isolines of the integral $\int_m^{m'} (1-H/H_m)^{\frac{1}{2}} dl$, namely J/w. The integration is along a line of force between the mirror points m, m'. The isolines —— and ---- refer respectively to mirror points where H is 0·45 G or 0·20 G. The values of J/w indicated on the isolines are lengths in terms of the earth's radius as unit. (After Vestine & Sibley 1959.) (See § 6 (*b*).)

(*c*) *Storm–time variation of the auroral zones*

(i) *Equatorward shift of auroras*

One of the remarkable features of large magnetic disturbances is the shift of auroras towards the equator. Diffuse red auroras are seen from geomagnetic latitudes as low as 30° during the largest magnetic storms. Here, however, we are concerned with the equatorward shift of auroras with arc structure (e.g. rayed arcs or rayed bands).

In figure 6 we have seen that when H_{R0} is as large as 500 γ, the neutral line advances towards the earth as near as 3·6 earth radii from the earth's centre. Then the rayed arcs or bands are expected to appear at 58° 11′ geomagnetic latitude. This also depends on the position of the ring current. For $(q/a) = 4$, $(b/a) = 0{\cdot}5$ and $(A/a) = 4{\cdot}5$, the position of the neutral line is about 3·1 earth radii, which corresponds to 55° 30′ geomagnetic latitude. If we take into account the distortion of the earth's field, the neutral line may advance further towards the earth.

Figure 17, plate 1 shows an example of a rayed band. It appeared in the northern United States on the night of 23 September 1957, during one of the largest magnetic storms that occurred during the I.G.Y. This band was seen (*a*) north of Pullman (geomag. lat. 53·5° N), and (*b*) overhead at Choteau (geomag. lat. 55° N). These two stations are separated from each other by about 250 miles in longitude and are about 10° south of the auroral zone (see also figure 10 and table 1). An intense X-ray burst was observed by Winckler (1960) at

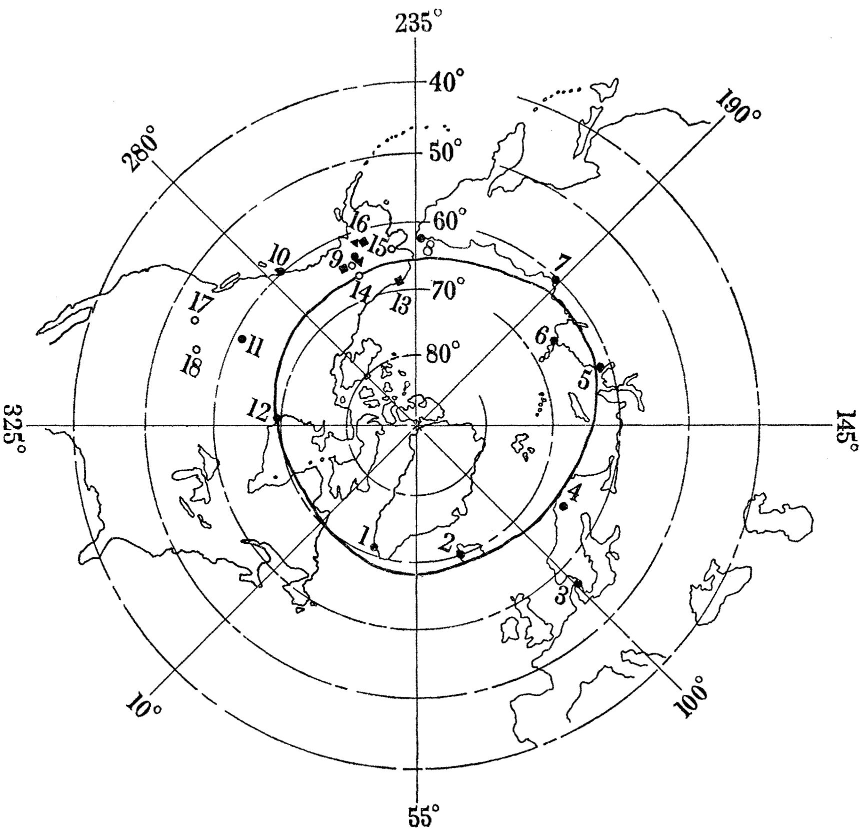

Figure 10. Map showing the positions of the stations whose data are used in the text. Vestine's isochasm for the maximum auroral frequency is also shown. (See §§ 6 (*c*) and 9.) (●, Magnetographs; ○, all-sky camera; v.h.f., auroral radar; △, riometer.)

1. Julianehaab	2. Reykjavik	3. Rude Skov	4. Abisko	5. Dixon Is.	6. Chelyuskin
7. Tixie Bay	8. Cape Wellen	9. College	10. Sitka	11. Meanook	12. Churchill
13. Point Barrow	14. Fort Yukon	15. Kotzebue	16. Farewell	17. Pullman	18. Choteau

Minneapolis (geomag. lat. 55° N), further to the east, at the same time. This time was near local dawn, but observations suggested a further extension of the rayed bands towards the east.

Barbier (1958) and Roach (1959, private communication) reported that at the same time a monochromatic 6300 Å arc detached itself from the rayed band and moved southward (see also Roach & Marovich 1959). The band also extended to the west, at least as far as Alaska (Akasofu 1960 *b*).

(ii) *Day-to-day variation of the auroral zones*

After the ring current attains its maximum intensity, when also we may expect that the aurora extends furthest towards the equator, the ring current begins to decay. The rapidity of decay first increases and then slows down. Correspondingly the neutral line will recede from the earth, and the auroral arc will return polewards. As has been seen in § 6 (*a*), there is a daily variation of the position of the auroral zones. Also, in the northern hemisphere, the southernmost latitude of the aurora increases from night to night, as the DR intensity becomes less and less. During this interval, we might also expect a repetition of similar auroral displays on two or three consecutive nights.

TABLE 1. LIST OF THE STATIONS FROM WHICH DATA ARE MENTIONED IN THE TEXT

	geographic		geomagnetic	
	latitude	longitude	latitude	longitude
1. Julianehaab	60° 43′ N	46° 02′ W	70·1° N	35·4°
2. Reykjavik	64° 11′ N	21° 42′ W	70·2° N	71·0°
3. Rude Skov	55° 51′ N	12° 27′ E	55·9° N	98·6°
4. Abisko	68° 21′ N	18° 49′ E	66·1° N	115·0°
5. Dixon Is.	73° 30′ N	80° 25′ E	63·0° N	161·5°
6. Chelyuskin	77° 43′ N	104° 17′ E	66·3° N	176·5°
7. Tixie Bay	71° 35′ N	129° 00′ E	60·4° N	191·4°
8. Cape Wellen	66° 10′ N	169° 50′ W	61·8° N	237·0°
9. College	64° 51′ N	147° 50′ W	64·7° N	256·5°
10. Sitka	57° 03′ N	135° 20′ W	60·0° N	275·4°
11. Meanook	54° 37′ N	113° 20′ W	61·8° N	301·0°
12. Churchill	58° 48′ N	94° 12′ W	68·7° N	322·7°
13. Point Barrow	71° 18′ N	156° 46′ W	68·6° N	241·0°
14. Fort Yukon	66° 34′ N	145° 18′ W	66·7° N	256·8°
15. Kotzebue	66° 53′ N	162° 36′ W	63·6° N	242·2°
16. Farewell	62° 32′ N	153° 54′ W	61·4° N	253·4°
17. Pullman	46° 43′ N	117° 10′ W	53·5° N	300·6°
18. Choteau	48° N	112° W	55° N	305°
19. San Juan	18° 23′ N	66° 07′ W	29·9° N	3·2°
20. Tamanrasset	22° 48′ N	5° 31′ E	25·4° N	79·6°
21. M'Bour	14° 24′ N	16° 57′ W	21·2° N	55·1°
22. Elisabethville	11° 38′ S	27° 25′ E	12·7° S	94·1°
23. Hermanus	34° 25′ S	19° 13′ E	33·3° S	80·3°
24. Muntinlupa	14° 22′ N	121° 01′ E	3·1° N	189·7°
25. Kakioka	36° 14′ N	140° 11′ E	26·0° N	206·0°
26. Guam	13° 27′ N	144° 45′ E	3·9° N	212·8°
27. Honolulu	21° 18′ N	158° 06′ W	21·0° N	266·4°
28. Apia	13° 48′ N	171° 46′ W	16·0° S	260·2°

DESCRIPTION OF PLATE 1

FIGURE 17. All-sky camera photographs of the aurora of 23 September 1957, from the U.S. stations (*a*) Pullman (geomag. lat. 53·5° N) at 1052 G.M.T., and (*b*) Choteau (geomag. lat. 55° N), about 400 km to the east of Pullman, at 1054 G.M.T. The wavy rayed band here appears about 10° south of the auroral zone. A great magnetic disturbance was in progress at this time. (See §§ 6 (*c*) and 10 (*a*).) I.G.Y. observers: J. G. Robison (Pullman) and L. B. Craine (Choteau).

FIGURE 18. All-sky camera photograph taken from Fort Yukon, Alaska (geomag. lat. 60·7° N) at 0937 G.M.T. on 16 February 1958; the local time was 2337, 15 February. The photograph shows at least five separate arcs. They extend in the geomagnetic east–west direction. (Geophysical Institute, Alaska.) (See §§ 7 (*a*) and (*e*).)

FIGURE 19. All-sky camera photograph taken at Kotzebue, Alaska (geomag. lat. 63·6° N) at 1119 G.M.T. 23 September 1957 (0019 local time). It shows an exceptionally fine example of a rayed arc. A large polar magnetic disturbance of order 1500γ was observed at the same time at College, Alaska. (Geophysical Institute, Alaska.) (See §§ 8 (*a*) and 10 (*a*), (*b*).)

FIGURE 24. All-sky camera photograph taken from Kotzebue, Alaska (geomag. lat. 63·6° N) at 1053 G.M.T. 22 September 1957 (local time, 2353 21 September 1957). Irregularly folded rayed curtains extend in the north–south direction near the horizon on the east and the west. (Geophysical Institute, Alaska.) (See § 10 (*a*).)

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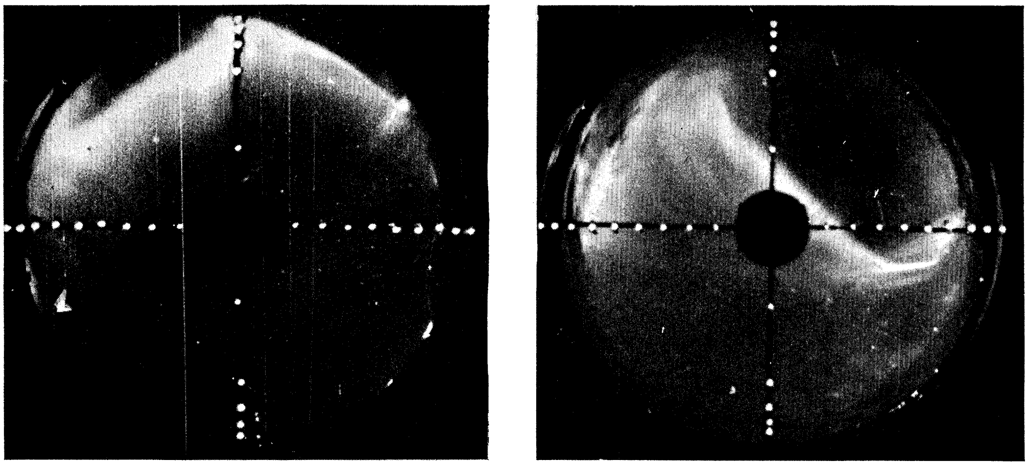

(*a*) S S (*b*)

Figure 17

N N

W E

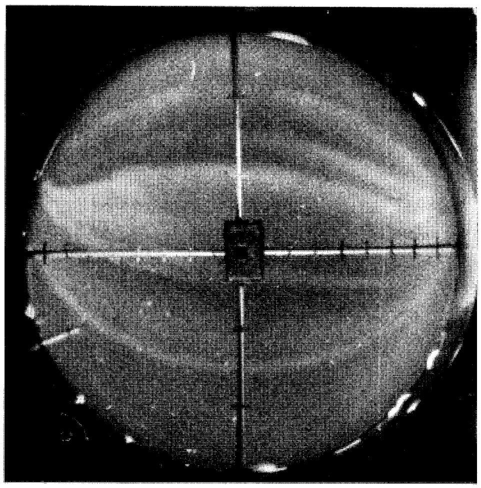

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Figure 18

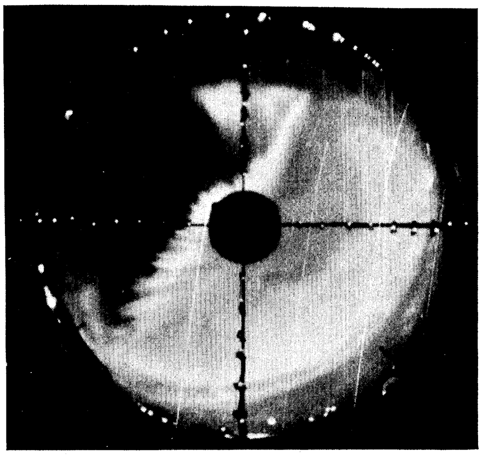

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Figure 19

N

W E

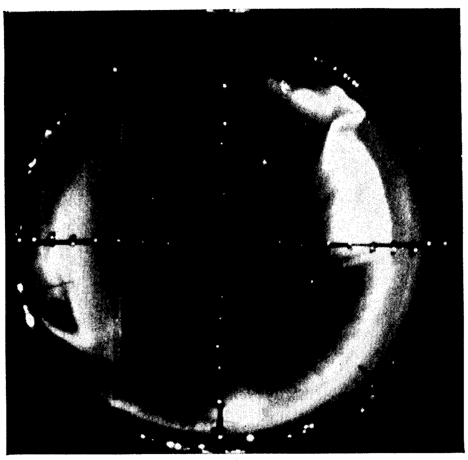

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Figure 24

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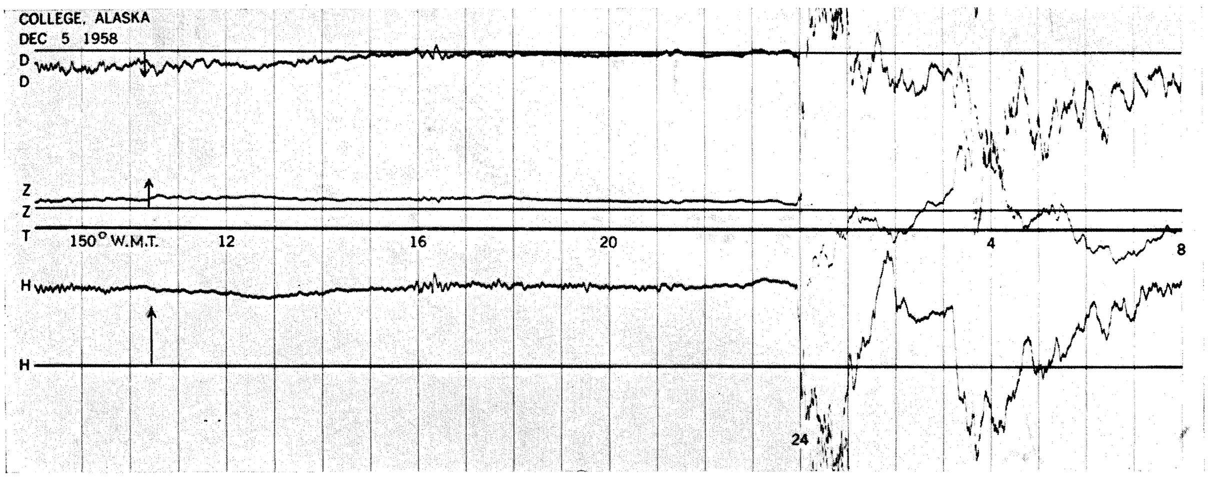

FIGURE 20. The College (Alaska) magnetogram of 5 to 6 December 1958. The hour marks refer to standard time of the meridian 150° W. Two small polar magnetic disturbances are shown, which began respectively at the local times 2357 and 0315. The arrows indicate the direction of increase of each component; their length corresponds to a variation of 100γ. The horizontal lines marked *H*, *D*, *Z* are the base lines from which the trace of the corresponding element is measured. (See § 9 (*c*).)

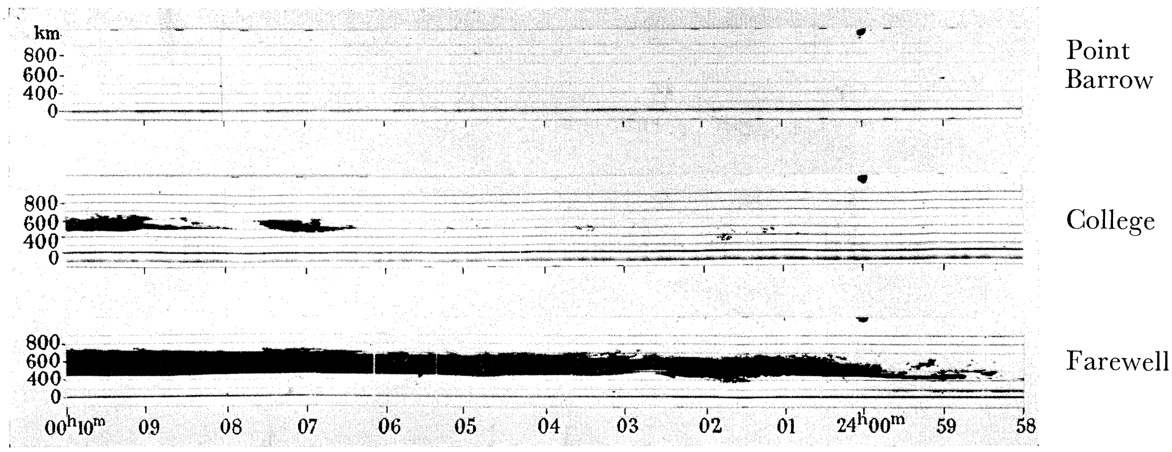

FIGURE 23. V.h.f. auroral radar data taken at Point Barrow (Alaska) (geomag. lat. 68·6° N), College (Alaska) (geomag. lat. 64·7° N) and Farewell (Alaska) (geomag. lat. 61·4° N) on the night of 5 to 6 December 1958. Note the radar echo at Farewell corresponding to the first polar disturbance in figure 20 (or the first break-up of the aurora in figure 21). (Leonard 1960, private communication). (See § 9 (*c*).)

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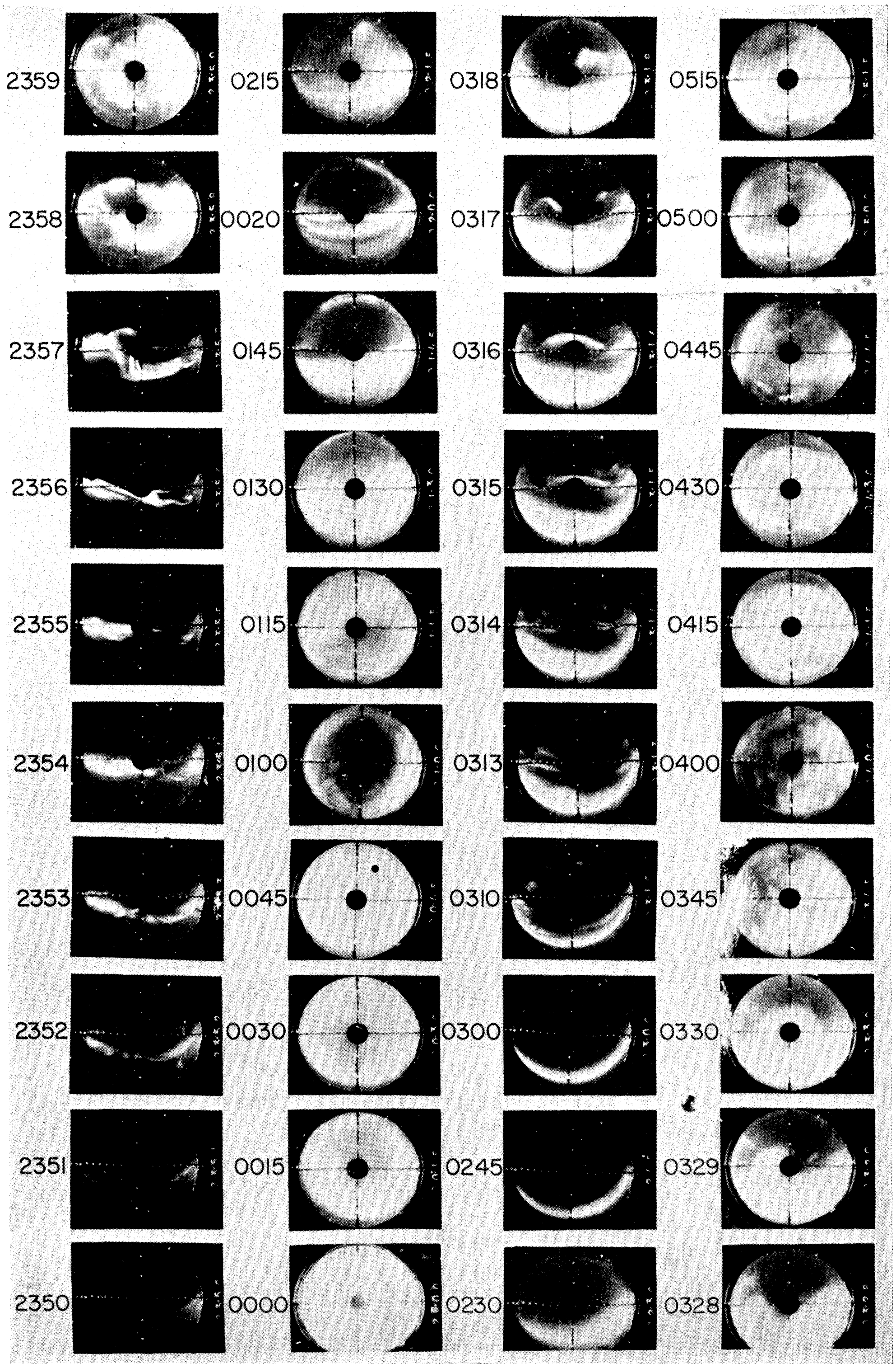

FIGURE 21. Some College (Alaska) all-sky camera photographs for the interval from 0950 to 1515 G.M.T. 6 December 1958 (local time, from 2350 5 to 0515 6 December 1958). This interval includes the two break-ups (at 2357 and 0315) associated with the two small polar magnetic disturbances shown in figure 20. (Geophysical Institute, Alaska.) (See § 9 (*c*).) In each photograph North is uppermost and East is on the right. The time marked 0020 should be 0200.

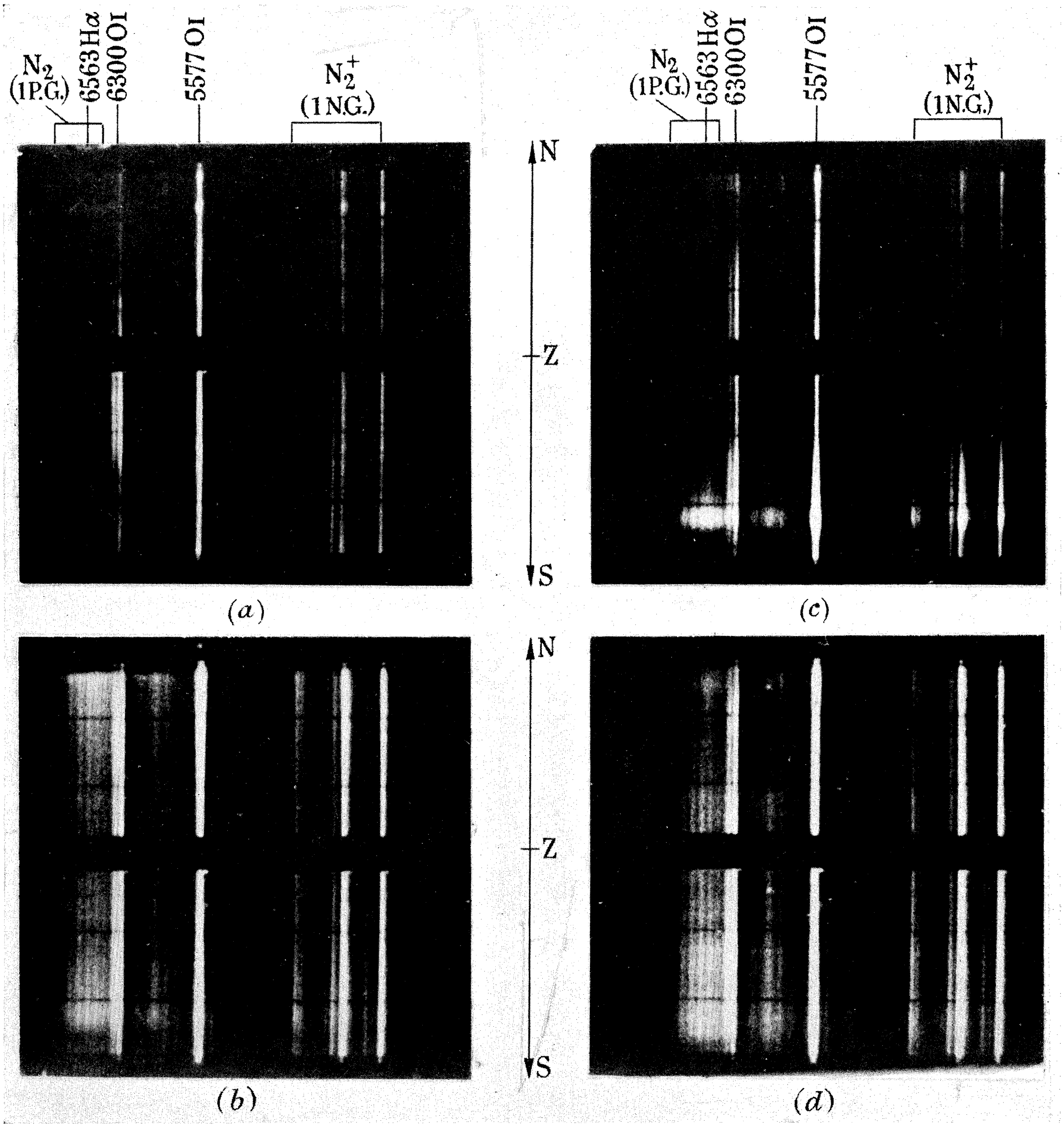

FIGURE 22. Four spectrograms taken at College, Alaska, on the night of 5 to 6 December 1958. Two of them, (*a*) and (*c*), were taken just before the auroral break-ups shown in figure 21. The others, (*b*) and (*d*), were taken just after the break-ups. (Rees 1960, private communication.) (See § 9 (*c*).)

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After a few nights, the aurora does not appear to progress appreciably equatorward and does not show any distinguished display; finally it disappears. This may correspond to the decay of the ring.

The lifetime of the neutral line depends on the nature and amount of variation of the ring current. It seems likely that the ring current is always present during the sun's active period, perhaps with a field of central intensity about 50 γ, although it may be too weak to produce neutral lines. Complexities arise if a new reinforcement of the ring current occurs during the decay period.

7. Particle injection associated with arcs

(a) *Diffuse quiet arcs, single or multiple*

At College, Alaska (geomag. lat. 64·7° N), auroral displays begin with a rather homogeneous arc near the poleward horizon. This is only a rough description, because the aurora is far distant ($>$ 300 km), and its details cannot be seen. When the arc advances southward, and can be seen more clearly, we often find that its brightness is continuously changing. At that stage ray structure is absent, or weak and rather diffuse. Figure 18, plate 1 shows the all-sky camera photograph taken from Fort Yukon, Alaska (geomag. lat. 66·7° N) at 0937 G.M.T. on 16 February 1958; the local time was 2337 on 15 February. The photograph shows at least five separate diffuse arcs. Two of them are south of the zenith. They extend in the geomagnetic east–west direction.

(b) *Auroral spectra and hydrogen emission*

The most prominent and common lines and bands found in the spectra of high latitude auroras are as follows. Atomic lines: wavelengths 5577 (OI), 6300 (OI), 6364 (OI), 3466 (NI) 5200 (NI), 6563 (HI) (= Hα), 4861 (HI) (= Hβ). Molecular bands = N_2 first positive, N_2 Vegard–Kaplan, N_2^+ first negative (including 3914 and 4709), N_2^+ Meinel bands and others. The identification of the lines and bands, and their excitation mechanisms, have been extensively studied by various workers. The results are well summarized in articles by Chamberlain & Meinel (1954) and by Bates (1954, 1960).

It has been suggested at times that extra-terrestrial protons are perhaps the most important agencies, direct or indirect, in producing the auroral luminosity. Recent detailed studies have shown, however, that the hydrogen lines appear most often in the spectra of quiet and diffuse forms, and less often in the active forms (cf. Romick & Elvey 1958; Galperin 1959; Montalbetti 1959). These studies suggest that there is no clear systematic relation between the various auroral luminosities and the intensity of the hydrogen emission.

As a result of comparing the intensities of Hβ and of 4709, Omholt (1957) suggested that auroras are caused mainly by fast electrons rather than by protons. Following his discussion, Bates (1960, p. 320) concluded that protons cannot be the dominant cause of the auroral luminosity most commonly seen in the auroral zone (see also § 9). The proton flux is at most 10^8 cm^{-2} s^{-1}. Usually it is much less, of order 5×10^5 cm^{-2} s^{-1}. The most probable proton energy may be of order 130 keV (cf. Bates 1954, p. 625).

(c) *Rocket observations*

In recent years rocket techniques for the exploration of the upper atmosphere have been extensively applied to the study of auroras. Meredith *et al.* (1958) showed that the energetic electron flux in a faint-rayed arc was confined to the region of auroral luminosity; protons were detected over a wider area bordering the auroras. McIlwain (1959) found that energetic electrons accounted for at least 90 % of the energy flux measured in the region of a diffuse auroral band penetrated by one of his rockets. These results support the conclusion derived from the spectroscopic studies (§ 7 (*b*)).

(*d*) *Theory of the stucture of a quiet arc*

In § 5 (*b*) we have discussed the motions of protons and electrons around the X-type neutral line. When there is no electric field along the neutral line, the width $2X_{1e}$ for 30 keV electrons is about one-tenth of that for 100 keV protons (see equations (55) and (56)). It has been shown that the electron strip is located in the middle of the proton strip (see figure 8 *b*). The protons and electrons in these strips are unstable, and travel to auroral levels in the atmosphere, along the lines of force through the strips.

If the strips are at 6 earth radii from the earth's centre (so that $H_M = 150\,\gamma$ in equation (58′)) the strip width $2X_{1p}$ and the radius of gyration R_p for 130 keV are given by

$$2X_{1p} = 2R_p = 695\,\text{km}.$$

The distance at the auroral level between the two lines of force which in the equatorial plane, at the distance of 6 earth radii, are separated by 695 km, is 29 km. This may correspond with the width of the layer of auroral emission of Hα and of the other lines excited by protons. In the middle part of this layer there may be a thin layer excited by electrons, which is supposed to produce the major part of the luminosity visually observed. The thickness of this inner layer is of order 3 km ($\sim 29/10$). This structure seems to agree well with the rocket observations of quiet forms of the aurora.

The fundamental form of the aurora, narrow in width but extremely long in the east–west direction, accords with figure 7, in which for a medium disturbance ($H_{R0} = 70\,\gamma$) the projection of the neutral line on the earth encircles nearly three-quarters of the auroral zone.

(*e*) *Multiplicity of arcs and the outer geomagnetic field*

As Elvey remarks (1957), the aurora in the simplest case consists of only one arc, but in many cases it is multiple, consisting of many arcs. The arcs are, in general, nearly parallel to each other. An example is shown in figure 11 (Elvey 1957, his figure 5). Figure 18, plate 1 gives another example.

This suggests, according to our hypothesis, that the outer geomagnetic field at the time of a magnetic storm is not at all simple. The irregular and intermittent injection of solar particles into the geomagnetic field may produce several current layers there. This would complicate the field distribution, and may produce several neutral lines. Table 2 shows the approximate geomagnetic latitudes of the arcs of figure 11 and the corresponding positions of neutral lines in the equatorial plane, calculated from (62). Figure 12 shows a tentative distribution of the corresponding magnetic field intensity along a radius from the earth, in the equatorial plane. The line AB represents the equatorial radius in the geomagnetic meridian half-plane that passes through College, Alaska. The graph represents schematically the magnitude and sign of the magnetic intensity along AB. The intensity scale is not defined. The dots on the line indicate points on X-type neutral lines of the field, corresponding to the arcs in figure 11, and are denoted by similar letters. The intervening zero points are on O-type neutral lines.

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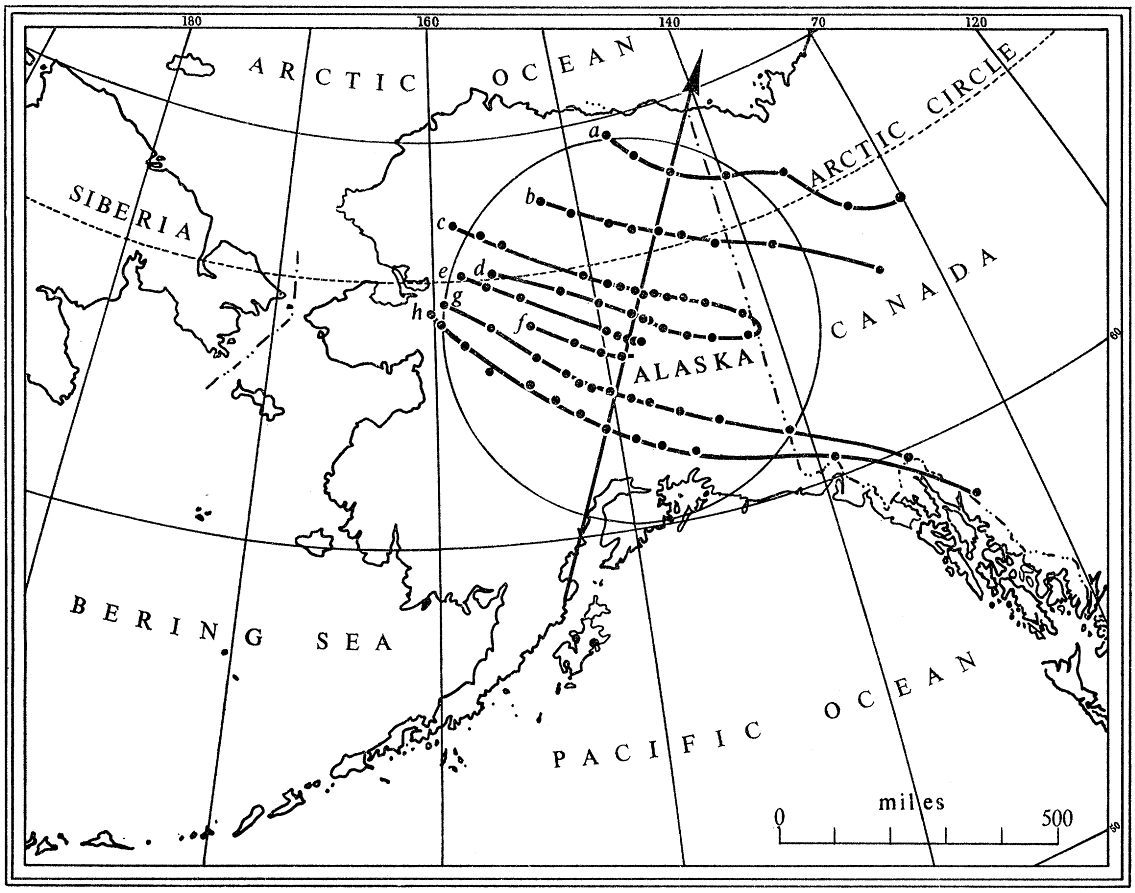

FIGURE 11. Ground-plan projection of auroral draperies over Alaska at 0734 G.M.T., 1 October 1954 (local time 2124 30 September 1954); from an all-sky camera photograph taken at College, Alaska. (After Elvey 1957.) (See § 7 (*e*).)

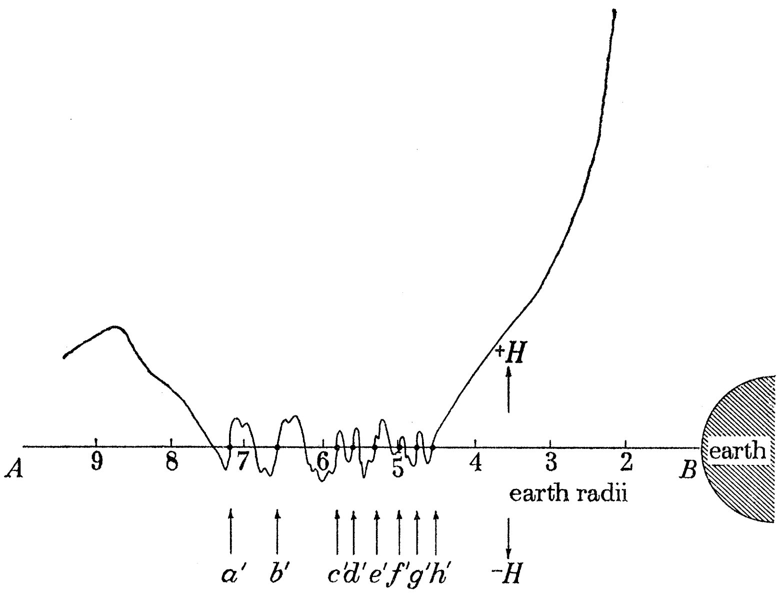

FIGURE 12. Schematic distribution of magnetic field intensity along the equatorial radius to which figure 11 refers, in the geomagnetic meridian half-plane that passes through College, Alaska. The dots on the line indicate points on *X*-type neutral lines of the field, corresponding to the arcs, in figure 11, denoted by similar letters. (See § 7 (*e*).)

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TABLE 2. POSITIONS OF THE ARCS IN FIGURE 11 AND OF THE CORRESPONDING X-TYPE NEUTRAL LINES

arc	position (geomag. lat.)	distance from the earth's centre (earth radii)
a	68° 06′	7·2
b	67° 06′	6·6
c	65° 24′	5·8
d	64° 54′	5·6
e	64° 12′	5·3
f	63° 18′	5·0
g	62° 42′	4·7
h	62° 00′	4·5

In this connexion we may refer to the observations of the density of the radiation belts made by the U.S. satellite Explorer VI. This had an elongated orbit extending out to 7·6 earth radii from the earth's centre; the orbital period is $12\frac{1}{2}$ h. Observations of particular interest were made during the storm that occurred from 16 to 18 August 1959. During the earlier part of this disturbance, a notable decrease in the counting rate in the outer radiation belt was reported by Arnoldy, Hoffman & Winckler (1960). Moreover, the same rocket revealed intense fluctuations of the rate between 5·5 and 7·5 earth radii from the centre of the earth (Rosen, Farley & Sonett 1960). This may have been associated with much complexity of the outer geomagnetic field, as suggested by the study of the multiplicity of arcs.

It is interesting to note that if a new current layer appears at 6·3 earth radii in figure 12 this changes the field distribution appreciably. The result is that the neutral lines c', d', e', f', g' and h' advance towards the earth and the neutral lines a' and b' recede from the earth. Thus the corresponding arcs c, d, e, f, g and h move southward as a group, and the arcs a and b move northwards. All-sky films often show such systematic group motions of arcs before break-up (cf. the recent work by Davis & Kimball (1960)).

The lifetime of each arc depends on the life of the irregular current layer. If there is some local action which infringes the conditions discussed in § 3 (a), the particles diffuse from one surface defined by $J = \int_{m'}^{m} w_s \, dl$ to another, and merge into the background ionization.

8. RAYED ARCS

(a) *Rayed arcs and high-energy electrons*

The most spectacular phase of auroral displays comes when the quiet form brightens and fine ray-structure appears in it. Figure 19, plate 1 shows an all-sky camera photograph of an exceptionally fine example of a rayed arc. The photograph was taken at Kotzebue, Alaska (geomag. lat. 63·6° N) at 1119 G.M.T., 23 September 1957 (0019 local time). A large polar magnetic disturbance (of order 1500 γ in the horizontal component) was recorded at College at the same time.

During this stage electrons make the major contribution to the auroral phenomena. This has been confirmed by balloon measurements (Winckler *et al.* 1958, 1959; Winckler 1960). The simultaneous appearance of strong auroral-associated radio absorption might also indicate the precipitation of high-energy electrons (Chapman & Little 1957; Little

& Leinbach 1958; Reid & Collins 1959). It is thought that the energy of the typical auroral electrons is of order 30 keV, and that their flux is of order $10^{10}\,\mathrm{cm^{-2}\,s^{-1}}$. The maximum radius of gyration of such electrons at the auroral level (taking the total intensity of the earth's field to be 0·6 G) is 7·1 m. We interpret the ray structure in the arc as an instability of the electron sheet-stream extending from the neutral line to the visible aurora. This is further discussed in § 10.

One of the outstanding facts to be explained is the extreme thinness of the rayed curtains and folds. Elvey (1957) estimated it to be not more than 250 m. In the meridian plane the two lines of force which in the auroral zone are separated by 250 m are only 6 km apart in the equatorial plane. This distance of 6 km may be contrasted with the width of the Van Allen radiation belts—of order at least 10 000 km. This suggests that the equatorial sources of auroral rayed forms are extremely local features in these immense belts. Similarly the width 6 km is far less than the radius of gyration (348 km) of the 130 keV protons, or than their mean free path, or than the wavelength of hydromagnetic waves (this is of order 200 km).

In § 5 (*b*) we found that an eastward electric field can accelerate the electrons in a strip around the neutral line of width $2X_{1e}$ and that it will produce an extremely large gradient of the magnetic field (see also figure 8 (*c*)). It seems likely that rayed arcs are produced by this strong constriction of electrons around the neutral line due to the electric field proposed in § 5 (*b*). The substantial increase of electron flux at this stage supports this view. As before, in § 7 (*d*), taking H_M to be $150\,\gamma$, the width of the region traversed by the 30 keV electrons drifting along the neutral line is

$$2X_{1e} = 2R_e = 5{\cdot}6\,\mathrm{km}.$$

This agrees well with the above estimate of 6 km. The thickness of the corresponding arc would be 230 m.

This thinness, of order less than 500 m, may also be contrasted with the width of the zone in which there is a continual flux of high-energy electrons. This zone is at least 1000 km wide (from geomag. lat. 65° to 75°) (Van Allen 1957; Vernov *et al.* 1959). This continual flux may be connected with the high intensity of the 5577 Å glow in the auroral zone (Roach & Rees 1960). But in the rayed arc there must be a far stronger concentration of electrons.

The width of the equatorial exit band (proton flow) decreases from 695 km during quiet arcs to 5·6 km at the time of the electron constriction. The motion of protons in a field with such a steep gradient may be unstable, and the constriction of protons there may be much less than that of electrons. An abrupt diminution of Hα radiation at the time of auroral break-up has in fact been reported by many workers (Dahlstrom & Hunten 1951; Fan & Schulte 1954; Romick & Elvey 1958; Galperin 1959; Fan 1958; Montalbetti 1959; Malville 1959*a*).

(*b*) *Runaway electrons near the neutral line*

In § 5 (*b*) we have introduced the hypothesis of an electric field along the neutral line, that accelerates and constricts the electron flow. In § 8 (*a*) we have seen how rayed arcs can be explained by this electron flow. In their most active stage, the height of the lower border of the rays is decidedly less (from 70 to 90 km; cf. Elvey 1957) than for quiet arcs (around 120 km); also the lower border of the rays is purplish red. Auroras showing this purplish red border are said to be of type B. By using Bates's table 12 (1954, p. 625), Malville (1959 b) inferred that type B aurora is caused primarily by high energy electrons (of order 250 keV). (This is because the atmospheric density rapidly increases downwards.) Therefore a considerable acceleration of the electrons seems to accompany the sudden change of auroral form from quiet to the most active.

In this section we associate the acceleration due to the electric field near the neutral line with the 'runaway' phenomenon. When an electric field E is sufficiently weak (and the deviation from the Maxwellian velocity distribution is small), the electric current produced is proportional to the applied electric field. However, if an electric field exceeds a critical value, denoted by E_c, the electric current j increases more than proportionally to E, and there is no time-independent relation between them. This is called the 'runaway' phenomenon.

The critical electric field E_c for which runaway can occur has been given by Dreicer (1959) as follows

$$\left.\begin{aligned} E_c &= \frac{e}{\lambda_D^2} \ln (\lambda_D/p_0) \text{ e.s.u.,} \\ &= 1{\cdot}44 \times 10^{-6}/\lambda_D^2 \text{ V/cm.} \end{aligned}\right\} \tag{63}$$

Here λ_D denotes the Debye length

$$\lambda_D = \left\{\frac{\kappa T_e}{4\pi n_e e^2}\right\}^{\frac{1}{2}} \text{cm;} \tag{64}$$

also $\ln(\lambda_D/p_0)$ is the cut-off factor, here taken to be 10 for all conditions.

Even if we take the background thermal electrons (near the neutral line) to have energy as low as 1 eV ($T = 7734$ °K) and if the density is as high as 10^2/cm^3, the critical field E_c is only of order $3{\cdot}9 \times 10^{-10}$ V/cm ($\lambda_D = 61$ cm). For more energetic electrons E_c will be still less. The runaway effect may be experienced by both types of electron in the belt. The electrons that produce diffuse arcs must have energy at least of order 10 keV. They must be accelerated in order to produce type B auroras (the E_c for energetic electrons is smaller than that for the thermal electrons).

We often observe a remarkable poleward motion of the arc when the arc brightens. This movement is simultaneous over a very extensive range of longitude along the auroral zone. The poleward speed of the auroral motion, of order 100 m/s, suggests transverse motion of the neutral line with a speed of order 2·4 km/s there.

The corresponding order of magnitude of the electric field may be estimated from the approximate formula

$$|E| \sim v \times H/c.$$

Taking $v = 2{\cdot}4$ km/s and $H = 5 \times 10^{-4}$ G, E is of order $4{\cdot}2 \times 10^{-9}$ e.s.u. $= 1{\cdot}2 \times 10^{-6}$ V/cm. This much exceeds the critical field $E_c = 3{\cdot}9 \times 10^{-10}$ V/cm for 'runaway' under the conditions that obtain there.

9. Polar magnetic disturbances

One of the most important associations with sudden changes of the auroral form is the simultaneous appearance of polar magnetic disturbances DP. As mentioned in § 2, strong electric currents—auroral electrojets—then appear at the ionospheric E level along the auroral zone.

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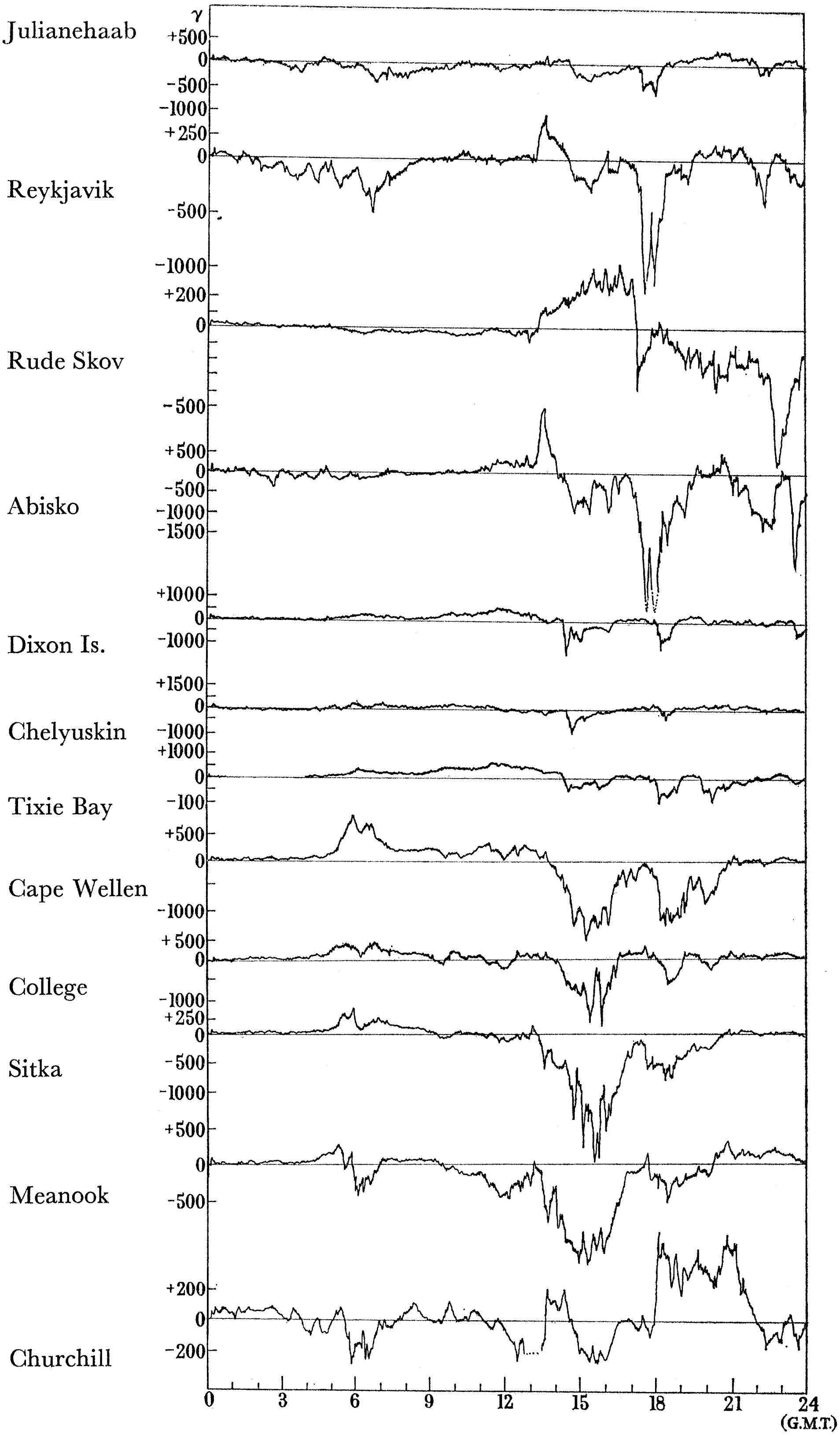

FIGURE 13. Horizontal component magnetograms of the storm of 29 September 1957 from twelve stations located along the northern auroral zone. The sudden commencement occurred at 0016 G.M.T. 29 September 1957. (See § 9.)

Like active auroras, DP disturbances are intermittent and sporadic, but are a fairly large-scale phenomenon. They are impulsive, and their lifetime is a few hours at most. Between the disturbances the magnetic condition is rather quiet. A quiet form of the aurora is often seen during this interval.

Figure 13 shows the horizontal component magnetograms of 29 September 1957 from 12 observatories, fairly well spaced along the northern auroral zone (see figure 10). The mean distance between adjacent observatories is about 1300 km. All the major DP's are likely to

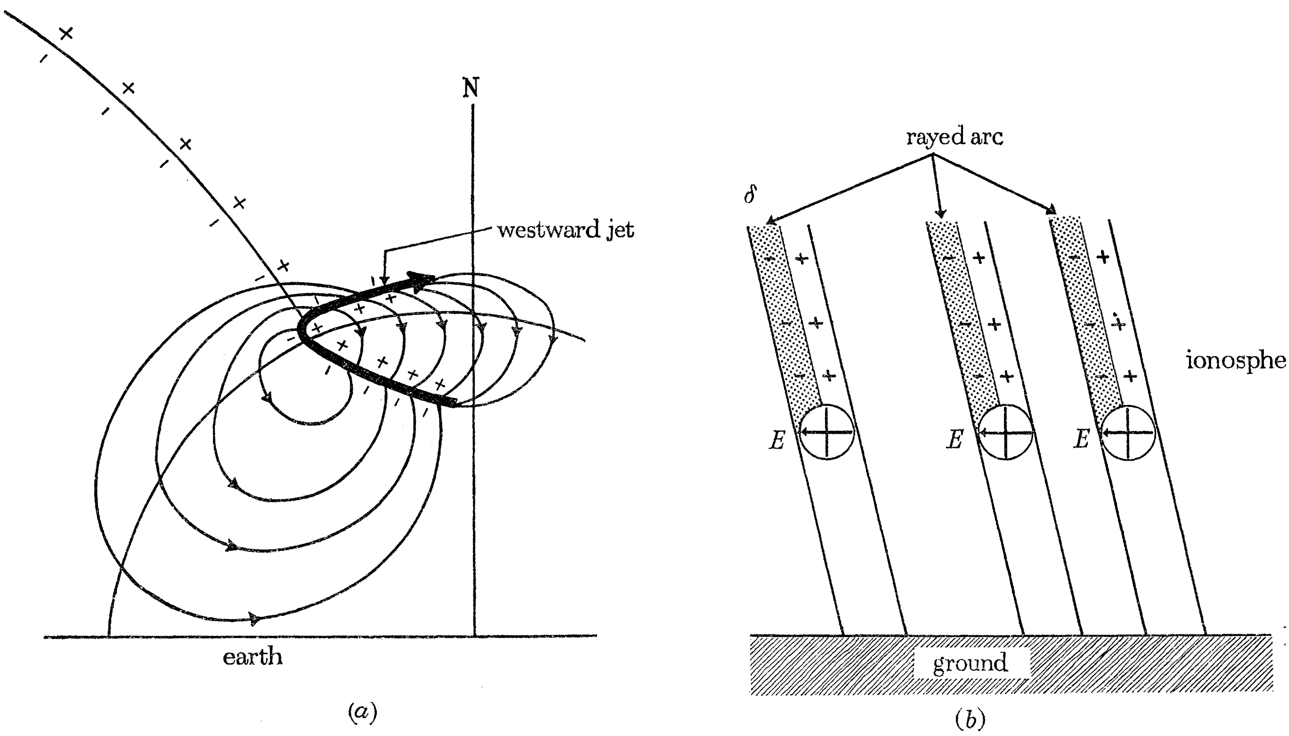

FIGURE 14. (*a*) Schematic perspective representation of part of the DP current system associated with a westward auroral electrojet. The electron and proton fluxes near a typical line of force ending on the electrojet are indicated. All the current lines and the electrojet (thick line) are located in the ionosphere, that is, on a spherical surface concentric with the earth's surface. (See § 9.)

(*b*) Schematic representation of three rayed arcs in the northern auroral zone showing, in meridian cross-section, the supposed distribution of the electron and proton fluxes. The equatorward electric fields in the ionosphere are indicated, and the cross-sections of the resulting westward auroral electrojets near the base of the arc are also shown. The cross inside each section signifies that the current is into the paper, that is, westward. (See § 9.)

be recorded in the figure. The sudden commencement occurred at 0016 G.M.T. 29 September. The magnetic field along the auroral zone remained rather quiet for nearly 14 h, until 1410. Then the first fairly large DP appeared (there was a small DP at about 0600 G.M.T.). The second DP (one of the largest that occurred during the IGY) began at about 1710 G.M.T. The third began at about 1805. It seems likely that active auroras were seen at these times along the dark part of the auroral zone, but data on this point are not yet available.

It should be particularly noticed that even during an intense magnetic storm, like that of 29 September 1957, the magnetic disturbances in the auroral zone are intermittent, with quiet periods between. This is the major reason why we have introduced the hypothesis of an intermittent electric field that activates the particles near the neutral line.

Figure 13 shows that the auroral jet currents were limited to certain longitude sectors of the auroral zone. They will tend to set up a polarization field at their eastern and western ends. This will cause the electric current to complete its circuit over the polar cap, and also in lower latitudes (see figure 14*a*). Thus changes of electromagnetic condition in the auroral zone are communicated even to the equatorial zone without delay (within the time accuracy of the observations). They often affect the equatorial electrojet in the ionosphere along the magnetic dip equator. This was first noticed by Sugiura (1953) as a daytime enhancement of sudden commencement at Huancayo (geomag. lat. 0·6° S). A detailed study of this phenomenon has recently been undertaken by Akasofu & Chapman. Thus the DP disturbance, originating in the polar regions, becomes a worldwide disturbance. Current diagrams for this kind of disturbance have been drawn by Chapman (1935); Vestine (1940) and Fukushima (1953).

(*a*) *The electric field of polar magnetic disturbances*

The impulsion of the auroral electrojets and their closing currents has been ascribed to dynamo action, such as produces the solar and lunar daily magnetic variations Sq and L. Such dynamo theories of DP have been developed and discussed most recently by Fukushima (1953), Vestine (1954) and Obayashi & Jacobs (1957). They assumed specially high conductivity along the auroral zone, and examined the resulting extra currents generated there by the wind system that produces Sq.

Their theory has been critically examined by Maeda (1959), who concluded that DP requires a quite different wind system from that for Sq. Akasofu & Chapman (1961*a*) argued that the quick reversal of the DP current direction sometimes observed during the brief interval of sudden commencement cannot be explained by a dynamo theory, because the large-scale wind system in the polar region cannot reverse its direction in a minute or so. Likewise Akasofu (1960*b*) pointed out that a sudden appearance of the large-scale eastward motion of the aurora, when a quiet form becomes active, cannot be due to the atmospheric wind, but may be due to drift of electrons caused by an electric field.

Taking the x and y axes in such a way that they coincide respectively with geomagnetic south and east, the electric current equation in the ionosphere may be written

$$\left.\begin{aligned} I_x &= K_{xx}E_x + K_{xy}E_y, \\ I_y &= -K_{xy}E_x + K_{yy}E_y, \end{aligned}\right\} \tag{65}$$

where K_{ij} is a tensor component of the height-integrated conductivity of the ionosphere. Akasofu (1960*b*) has shown that the current intensity in the westward auroral jet may be expressed by

$$I_y \simeq -K_{xy}E_x, \tag{66}$$

and he estimated the necessary electric field. This field produces an eastward drift of the aurora, shown by eastward motion of the electrons in the ionosphere, with speed v_e. This motion appears simultaneously with polar magnetic disturbances, and the speed is given (Martyn 1953) by

$$|v_e| \simeq \sin\alpha_e E_x c/F. \tag{67}$$

Here $\tan\alpha_e = \omega_e/\nu_e$; ν_e denotes the collision frequency of electrons with neutral atoms and ω_e denotes the angular gyro-frequency of electrons: F denotes the total intensity of the magnetic field. This is a more general expression of the first term of (9) in a partially ionized gas. In the E and F regions of the ionosphere $\omega_e/\nu_e \gg 1$, so that $\sin\alpha_e \sim 1$ and

$$|v_e| \simeq E_x c/F. \tag{68}$$

Therefore we can determine the electric field without knowing the conductivity K_{xy} (this depends on the electron density there). A reasonable estimate of the eastward speed of the aurora, obtained by radio methods (cf. Bullough, Davison, Kaiser & Watkins 1957; Nichols 1959) is about $v_e = 360$ m/s, so that E_x is $6{\cdot}67\times10^{-7}$ e.s.u. ($= 20$ V/km).

(*b*) *Equations of motion of the auroral particles*

Neglecting the displacement current ($\partial E/\partial t$) in the ionized gas, Parker (1957) rewrites equation (10) thus

$$\rho\,\mathrm{d}\boldsymbol{v}_n/\mathrm{d}t = -\nabla(p_n+p_m)+[(\boldsymbol{H}\,.\,\nabla)\,\boldsymbol{H}/4\pi]\,[1+(p_n-p_s)/2p_m], \tag{69}$$

and

$$\boldsymbol{v}_n = c(\boldsymbol{E}\times\boldsymbol{H})/H. \tag{70}$$

If there is any tendency towards a small charge separation, we must add the term due to the electric polarization field $\boldsymbol{E}$. We may rewrite (69) thus:

$$\rho_p\,\mathrm{d}\boldsymbol{v}_{np}/\mathrm{d}t = -\nabla(p_{np}+p_m)+[(\boldsymbol{H}\,.\,\nabla)\boldsymbol{H}/4\pi]\,[1+(p_{np}-p_{sp})/2p_m]+n_p e\boldsymbol{E}, \tag{71}$$

$$\rho_e\,\mathrm{d}\boldsymbol{v}_{ne}/\mathrm{d}t = -\nabla(p_{ne}+p_m)+[(\boldsymbol{H}\,.\,\nabla)\,\boldsymbol{H}/4\pi]\,[1+(p_{ne}-p_{se})/2p_m]-n_e e\boldsymbol{E}. \tag{72}$$

Here

$$\boldsymbol{E} = 4\pi e\int(n_p-n_e)\,\mathrm{d}V. \tag{73}$$

We notice here again that the ring current discussed in § 4 (*a*) and equation (23) is a special case of (69); the centrifugal force (outward from the earth) in the motion of the particles along the curved lines of force of the earth's field is completely balanced by the inward Lorentz force produced by the vector product between the ring current intensity and the earth's field, $\boldsymbol{j}\times\boldsymbol{H}$.

We have shown in § 4 (*a*) that the electrons drift slowly eastward and the protons drift westwards; both contribute to the westward ring current. We may suppose that the introduction of the electric polarization field $\boldsymbol{E}$ in the equations of motion (71) and (72) will slightly perturb the above steady eastward or westward motion of particles. The electric field must be eastward because the electrons move eastwards and the protons move westwards. Therefore the particles will be accelerated inward or outward from the earth, according to (70), (71) and (72).

We have seen that when ray structure appears in the arc the proton flux, of order $5\times10^5\,\mathrm{cm^{-2}\,s^{-1}}$, seems to be diminished by a large factor. At the same time the electron flux may be increased from a value such as $10^6 \sim 10^8\,\mathrm{cm^{-2}\,s^{-1}}$ (cf. Van Allen 1957) to $10^{10}\,\mathrm{cm^{-2}\,s^{-1}}$. The great difference of the fluxes now becomes important, namely $10^{10}\,\mathrm{cm^{-2}\,s^{-1}}$ for the electrons and perhaps $5\times10^3\,\mathrm{cm^{-2}\,s^{-1}}$ for the protons. The electrostatic unbalance produced by the large increase of the electron flux and the large decrease of the proton flux can, however, be kept to a low value by motion of the electrons in the flux regions themselves, and by a differential motion of the background ionization along the lines of force. This can keep the electric potential along the lines of force nearly uniform.

Hence $(n_p - n_e)$ and the electric field in (73) must be much less than that expected from the large difference between the proton and electron fluxes. The degree of neutrality along the lines of force depends on many factors, such as the density of electrons in the electron flux (from 10^{-2} to $1/\text{cm}^3$), the density of the background ionization (from 10^2 to $10^6/\text{cm}^3$), the free path of the background ionization (from $1{\cdot}8 \times 10^5$ to $2{\cdot}7 \times 10^{10}$ cm) or their collision interval (from 6×10^{-3} to 450 s), and also the characteristic time involved (from 0·1 to 10 s).

When the motion of the particles of the background is taken into account, another equation of motion must be added to (71) and (72). But the total amount of the background ionization is so large compared with the total number of particles in the fluxes that we may regard these as traversing nearly immobile background ionization. We also disregard any interaction between the fluxes and the background ionization. It is known, however, that an electron beam penetrating into a plasma produces plasma oscillation (cf. Gould 1960, p. 107).

As in (27′) the unperturbed east–west drift velocity due to the centrifugal force is given by

$$V = \frac{cm}{eHr_c} w_s^2. \tag{74}$$

Hence this motion is proportional to the individual energy of the particles. The ratio of the *total* energy of electrons in the electron flux to that of protons in the proton flux $(m_e n_e w_{se}^2 / m_p n_p w_{sp}^2)$ is as large as 2×10^4. As a consequence the electron flux may be nearly unaffected by the protons and may drift eastward in accordance with (74).

But the proton flux may be affected by the eastward electric polarization field (associated with a very small decrease in the energy of the electrons). As mentioned earlier (see § 4 (*a*)), the unperturbed state is in balance between the inward Lorentz force and the outward centrifugal force. Therefore any small change of the velocity of the westward drift motion of the protons by the eastward electric field breaks this balance. The reduction of the westward motion diminishes the inward Lorentz force (because of the reduction of j), so that the protons are accelerated outward from the earth according to (70) and (71).

This acceleration will produce outward motion of the proton layer and will separate the proton layer slightly from the electron layer. This can also be seen from equations (21) and (22), because $\boldsymbol{u}_n \cdot \boldsymbol{E} < 0$. However, such separation is checked by the new inward electric field (directed towards the earth) produced by this charge separation of the proton and the electron layers. If the westward motion of protons is completely halted by the eastward electric field, the outward centrifugal force must be balanced by this inward electric force, because the Lorentz force becomes zero and the motion of protons along the lines of force may not be affected by the eastward electric field (the centrifugal force is still operative). Thus, in this case the inward electric field between the proton and electron layers must satisfy

$$(en_p)\,(m_p w_{sp}^2/eHr_c)\,(\boldsymbol{r}_0 \times \boldsymbol{H}_0) \times \boldsymbol{H} - n_p e\boldsymbol{E} = 0, \tag{75}$$

where

$$\boldsymbol{j} \times \boldsymbol{H}/c = (en_p)\,(m_p w_{sp}^2/eHr_c)\,(\boldsymbol{r}_0 \times \boldsymbol{H}_0) \times \boldsymbol{H} = \text{the centrifugal force}. \tag{76}$$

Therefore the intensity of the electric field E is

$$E = m_p w_{sp}^2 / er_c. \tag{77}$$

When the protons and electrons descend to the auroral level, the protons appear just north of the rayed arc produced by the intense electron precipitation (but the reduction of the proton flux makes it difficult to observe the corresponding luminosity).

At the auroral level we take $\phi = 66° 30'$. Then (25) and (62) give $r_c = 1\cdot87 \times 10^9$ cm. For 130 keV protons, w is $4\cdot99 \times 10^8$ cm/s, so that the electric field is of order $2\cdot3 \times 10^{-6}$ e.s.u. As mentioned earlier, in § 9 (*a*), Akasofu (1960 *b*) infers that the westward auroral electrojet is produced by an equatorward electric field E_x of order $6\cdot7 \times 10^{-7}$ e.s.u. (= 20 V/km). (The conductivity K_{xy} is negative in the southern hemisphere.) This number agrees with the above value of $2\cdot3 \times 10^{-6}$ e.s.u. within a factor of 3.

Therefore it seems that a strong equatorward electric field appears along the active rayed arc, which drives strong westward currents and causes ionospheric electrons to drift eastwards. Figure 14 (*a*) and (*b*) shows this situation schematically.

The opposite case might occur in a rather quiet arc, if the proton flux exceeds the electron flux. In this case the electron layer is separated from the proton layer and moves outward from the earth, so that a northward electric field and an eastward auroral electrojet would be expected. Geomagnetic studies show that positive bays (caused by an eastward electrojet) appear sometimes in the early evening. But they are more rare and less intense than westward jets.

(*c*) *The polar magnetic disturbances of* 5 *to* 6 *December* 1958

In this section we discuss in some detail two typical (though small) polar magnetic disturbances which occurred during the night from 5 to 6 December 1958 (150° W.M.T.). The data we here present are the magnetogram, all-sky camera photographs, spectrograms, v.h.f. auroral radar and riometer data. They were mostly obtained by the Geophysical Institute, Alaska, as part of the I.G.Y. programme.

Figure 20, plate 2, shows the College magnetograms of 5 to 6 December 1958 (Alaskan Standard Time, 150° W.M.T.). Two polar magnetic disturbances began at 2356 and 0315 respectively. They are most easily seen in the *H* trace. It may be noticed how suddenly they began. Figure 21, plate 3, shows the all-sky camera photographs during this night. At 2350 the all-sky photograph shows at least three rather quiet arcs. Rees, Belon & Romick (1960) showed that the Hα emission layer had moved southward rapidly after a rather gradual daily southward motion in the early evening. The arcs slowly brightened between 2350 and 2355. At 2357 their intensity increased suddenly, and a polar magnetic disturbance began simultaneously. Until 0130 the auroras were too bright for any detailed structure to be seen on the all-sky camera photographs. After 0130 the auroras became quieter and the polar magnetic disturbance ceased. Between 0130 and 0315 several quiet arcs or bands are shown by the all-sky photographs.

At 0300 a bright band is shown in the southern sky. At 0310 the aurora became active again and began to move northward. At 0316 the band crossed overhead, and a polar magnetic storm also began simultaneously. The auroral display lasted until dawn, when twilight obscured it.

Figure 22, plate 4, shows the spectrograms taken at College on the same night (Rees 1960, private communication). Their exposure time was 15 min. Before the break-up of the aurora, Hα emission is shown in two spectrograms, taken from 2345 to 0000 (*a*) and from 0300 to 0315 (*c*). After the break-up, various lines and bands are enhanced in (*b*) and (*d*), taken from 0015 to 0030 and 0330 to 0345 respectively. It should be noticed that there seems no particular enhancement of the hydrogen emission after the break-up, although the contamination due to the first positive band of N_2 obscures it. This supports the view given in §§ 7 (*b*) and 8 (*a*) that the major luminosity of the aurora is produced by high-energy electrons coming down to auroral levels.

Figure 23, plate 2, shows the v.h.f. auroral radar data from Point Barrow (geomag. lat. 68·6° N), College and Farewell (geomag. lat. 61·4° N), from 2357 to 0010 on the same night (Leonard 1960, private communication; see also Leonard 1959). Owing to the geometrical complexity of the reflexion of the radio waves, discussed by Chapman (1952), detailed discussion must await further analysis of the data. However, it may be said that at 0010 the northernmost position of the aurora was at least to the north of the Alaskan coast of the Arctic Sea.

It is interesting to estimate the current density and the electron density in this aurora. Let us consider, as an example, the DP that began at 2357, 5 December 1958. The maximum change $\Delta H'$ may be estimated to be 300γ, so that the actual change ΔH due to the external field may be 200γ $(=\frac{2}{3}\Delta H')$. Assuming that this disturbance is produced by current flowing in two (closely spaced) overhead arcs, and that each current sheet has the width 2δ, ΔH is thus given by Chapman (1951)

$$\Delta H = 2\left(2I_y c \tan^{-1}\frac{2\delta/h}{1-(\delta/h)^2}\right). \tag{78}$$

Here I_y denotes the current density; h denotes the height of the jet, and is taken to be 120 km. As discussed above, we may take $2\delta = 500$ m for the width of the rayed arc, so that the above equation is approximated by

$$\Delta H \simeq 2\{2I_y c(2\delta/h)\}. \tag{79}$$

Putting $\Delta H = 2\times10^{-3}$ G $(=200\gamma)$, $2\delta = 500$ m, and $h = 120$ km, I_y is estimated to be $3{\cdot}6\times10^9$ e.s.u./cm $(=1{\cdot}2\times10^{-1}$ e.m.u./cm $= 1{\cdot}2$ A/cm). The total current J_y is, therefore, $1{\cdot}2$ A/cm $\times$ 1 km $= 1{\cdot}2\times10^5$ A.

For an equatorward electric field $E_x = 6{\cdot}67\times10^{-7}$ e.s.u., the conductivity K_{xy} is

$$K_{xy} = |I_y/E_x| = 5{\cdot}4\times10^{15}\ \text{e.s.u.} \quad (=6{\cdot}0\times10^{-6}\ \text{e.m.u.}). \tag{80}$$

Chapman (1956) gives the value of the Hall conductivity

$$k_2 = 4{\cdot}54\times10^{-15}\ \text{e.m.u.} = 4{\cdot}09\times10^6\ \text{e.s.u.}$$

at the height 125 km. This corresponds to an electron density n_e of $1{\cdot}5\times10^5/\text{cm}^3$. In the polar regions,

$$\int k_2\,\mathrm{d}h = K_{xy}.$$

For a thickness of 30 km, K_{xy} is $4{\cdot}09\times10^6$ e.s.u. $\times\,3\times10^6$ cm $= 1{\cdot}23\times10^{13}$ e.s.u. As the conductivity is linearly dependent upon the electron density, the electron density in the arc is given by

$$1{\cdot}5\times10^5\left(\frac{5{\cdot}4\times10^{15}}{1{\cdot}23\times10^{13}}\right) = 6{\cdot}6\times10^7/\text{cm}^3.$$

Several authors have discussed the electron density in the aurora (cf. Brown & Lovell 1958, p. 173; Chamberlain 1958, p. 191). It seems that the density of $7\times10^7/\text{cm}^3$ is possible in the narrow arc, where high energy electrons are entering at the rate $10^{10}\ \text{cm}^{-2}\,\text{s}^{-1}$.

Such an intense ionization may correspond to the sporadic E layer, which often accompanies an auroral display. The equatorward electric field E_x of order 20 V/km may also have an important relation to ionospheric disturbances in the polar region. But important data usually provided by ionospheric sounders are often lacking because of a complete or partial blackout. This is supposed to be due to the increase of ionization, especially in the lower part of the ionosphere, which absorbs the sounding radio wave (cf. Chapman & Little 1957). This absorption phenomenon has been demonstrated most clearly by the riometer observations (Little 1957).

Figure 15 shows the absorption of 27·6 Mc/s cosmic radio noise measured by the riometer at College, Alaska (geomag. lat. 64·7 °N) and Farewell, Alaska (geomag. lat. 61·4° N) during the night of 5 to 6 December 1958 (Leinbach 1960, private communication). It is interesting to note that the Z trace in figure 20 shows a negative excursion between 2337 and 0300, indicating that the westward auroral jet appeared north of College during that time. This seems to correspond with the larger absorption of the cosmic radio noise at College than at Farewell during this interval.

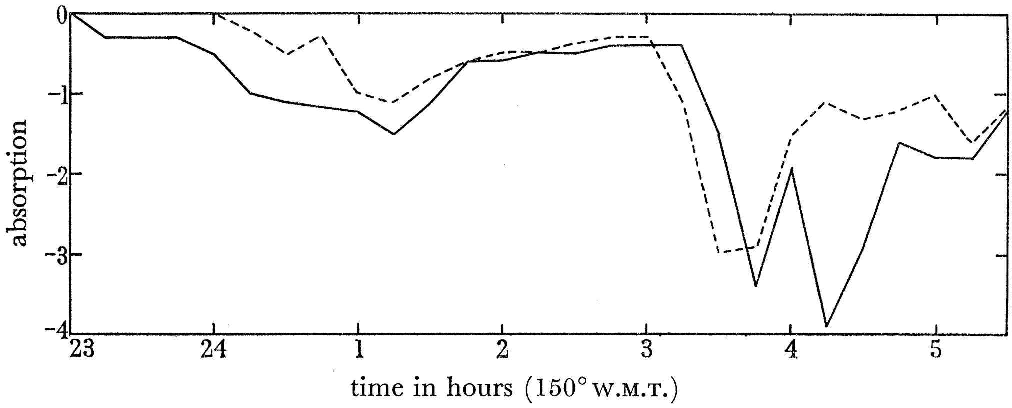

Figure 15. Absorption of 27·6 Mc/s cosmic radio noise at College, Alaska (geomag. lat. 64·7° N) (——) and Farewell, Alaska (geomag. lat. 61·4° N) (– – –), from 0900 to 1530 G.M.T. 6 December 1958 (local time from 2300 to 0530, 5 December 1958) (Leinbach 1960, private communication). (See § 9 (c).)

From 0300 to 0335 the jet was south of College. Correspondingly the absorption was larger at Farewell than at College. The all-sky camera photographs in figure 21 during this interval suggest that the bright band was the seat of the electrojet when it became active. Between 0335 and 0350 the jet was north of College, and then until 0415 it was south of College. After 0415 the jet was north of College again. After 0345 the absorption was larger at College than at Farewell. Hence it may be that except between 0350 and 0415, when the absorption was larger at College than at Farewell, the jet was north of College or overhead there, and when the absorption was larger at Farewell than at College, the jet was south of College.

10. Instabilities of auroras

(a) *Large-scale folded structure*

In this section we discuss the large-scale wavy or folded structure of active auroral arcs. Waviness begins to develop when the rayed arc becomes fairly active, and as the activity increases the waves often develop into folded structures. The 'wavelength' has a wide range, from 10 to 500 km. In their most developed form the folded structures or curtains often roll along. This stage is usually called 'auroral drapery'. At such times a purplish red colour may appear at the lower border of the arc. Figure 24, plate 1, shows an example of such draperies. The photograph was taken from Kotzebue, Alaska (geomag. lat. 63·6° N); the time was near local midnight (2353). Irregularly folded curtains extend in the north–south direction near the east and the west horizon. Figures 17 and 19 also show such examples of the drapery.

It is known that instability is one of the most common characteristics of strongly constricted electric currents. If strong constriction of an electric current occurs along the neutral line when the auroras become active, the current is likely to show such instability. Any distortion of the neutral line is immediately projected on to the auroral level, so that we regard wavy and folded structures of auroras as indicating a similar form for the associated neutral line or lines. If the purplish red colour mentioned above is due to high-energy electrons of more than 100 keV, as Malville (1959*b*) suggested, this supports the view that there is strong constriction and acceleration of electrons along the neutral line.

So far such a non-linear effect has been discussed only by perturbation methods (Lundquist 1951; Kruskal & Schwarzschild 1954; see also Pease 1958). Lundquist (1951) showed that kinks in a constricted cylindrical current in a free space develop when

$$H_\phi^2 > 2H_z^2,$$

where H_ϕ denotes the azimuthal field produced by the current itself and H_z denotes the original axial field. In our case, $H_z = 0$. But the boundary conditions are quite different from those considered by Lundquist. Hence his result cannot be applied immediately. But we suspect that the kinks develop in essentially the same way when the constriction of the current exceeds some critical value.

(*b*) *The ray structure* (*the small-scale zigzag structure*)

The auroral arc has another type of wave structure. Störmer gives a beautiful photograph showing this type (see Störmer 1955, his figure 83; the same photograph is reproduced by Chapman & Bartels 1940; plate 31*a*). A thin arc shows a small-scale zigzag structure when it becomes active. This produces an apparent inhomogeneity of the luminosity, vertical 'rays' or strips, as pointed out by Störmer (1955, pp. 325–327). Their width is less than 10 km.

This complicated feature can be most easily seen in an active corona such as appears when a rayed arc is seen at the magnetic zenith. In the most active coronas, extremely rapid and violent fluctuating motions of the zigzag structure are seen. Rapid changes of colour usually appear there.

The thin electron sheet-stream described in §§ 8 (*a*) and 9 (*b*) may be unstable in the sense discussed by Webster (1955, 1957) and demonstrated by his experiments. This instability is essentially due to the electrostatic interaction of the electrons in the electron sheet-stream. Small density fluctuations in the beam may produce the fluctuating electrostatic field E, expressed by (73), which produces the drift motion of electrons in the direction determined by (70)

$$\boldsymbol{v}_n = c(\boldsymbol{E}\times\boldsymbol{H})/H^2.$$

The rate of growth of such an instability has been discussed by Gould (1960, p. 100), who treated the problem as a small perturbation. The rate of growth is largest for a wavelength about eight times the beam thickness. In § 8 (*a*) we have shown that the thickness of the rayed arc is less than 500 m. Therefore the wavelength of the fine ray structure may be of order less than 4 km.

It may be noticed, however, that during the most active stage of the aurora there are various kinds of *apparent* motions that have never yet been properly described or recorded. They may be regarded as instabilities of the aurora. Such motions are usually accompanied by rapid change of colour, indicating quick changes in the excitation processes of various lines and bands of atoms and molecules at the auroral level.

(*c*) *The break-up of auroras*

Auroral displays in the final phase are often fantastic. Isolated rays or patches are scattered over the sky, and there is no longer any arc structure. In § 10 (*a*) we concluded that the wavy structures would develop into folded structures. We may regard this final display as the extreme case of the above instability. We may expect that when the folded form of the neutral line is completely developed, the configuration will be very complicated and tangled. High-energy particles produced there may be stored in some way in this tangled field and be released first from one place and then from another. The projection of this condition on the auroral level may result in isolated rays or pulsating patches. Isolated rays fixed in the sky for a few minutes are also often seen. This stage is associated with rapid magnetic fluctuations with periods from 20 s to 10 min. The ionospheric layers could not be the origin of these geomagnetic micro-pulsations (cf. Akasofu 1956), and we suggest that they correspond to irregular magnetic fluctuations around the neutral line. It should be noticed that at the same time strongly variable earth currents (Hessler & Wescott 1959), ionospheric absorption (Leinbach 1960, private communication) and luminosity from the entire sky (Murcray 1959) have been recorded, e.g. at College, Alaska. Recently Campbell (1960) reported simultaneous fluctuations of the auroral luminosity (3914 Å) and the magnetic pulsations.

This complicated stage may last until the irregular configuration of the field disappears and a rather simple neutral line is reformed. In fact, a quiet arc is often reformed after the break-up. An example is shown in figure 21, in which the quiet form is reformed between 0130 and 0310.

11. Polar magnetic disturbances (DP) and the ring current (DR)

This section describes some important features of individual magnetic storms that bear on auroral phenomena. A pair of highly contrasted magnetic storms is presented in figure 16. These storms began at 1626 G.M.T. 11 July and 0803 G.M.T. 15 July 1959. The ranges of their SSC's and DCF's may indicate the relative energies of the solar flux and its changes during the two storms. It seems likely that the rate of corpuscular flux on 11 to 12 July was at least as much as on 15 to 16 July.

However, the storms differ markedly in at least two ways. First of all, with respect to their DR part. The intensity of DR on 11 to 12 July was at most 30 γ and not well marked. But the DR on 15 to 16 July was about 450 γ and very clear.

Secondly, there was no clear DP on 11 to 12 July, but there were at least ten large DP pulses on 15 to 16 July. The 11 to 12 July storm is notable in the sense that a rather uniform

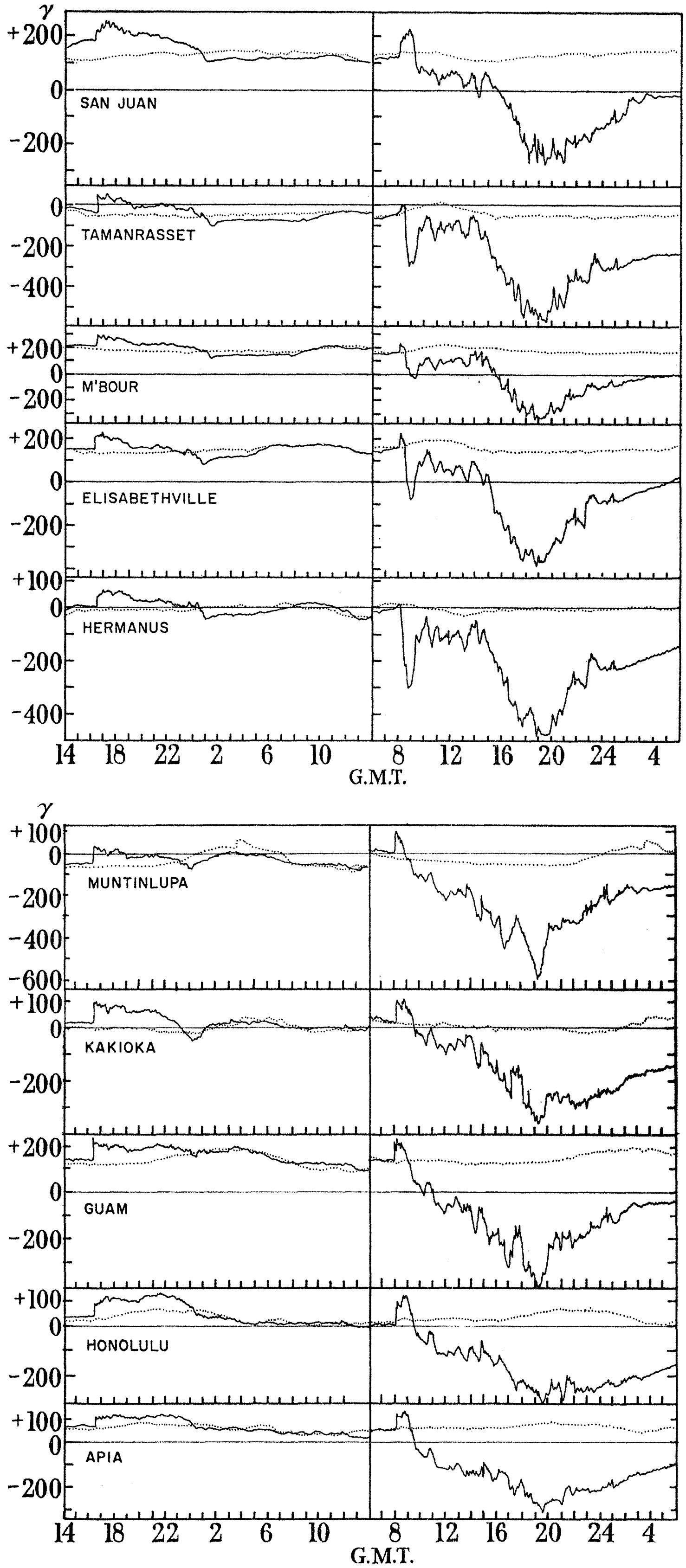

FIGURE 16. Horizontal component magnetograms of the storms of 11 to 12 July 1959 (left) and 15 to 16 July 1959 (right), from relatively low-latitude stations (between geomag. lat. 35° N and S). The stations are well distributed in longitude. The dotted lines show the record from 13 to 14 July 1959 (between the two storms) when the magnetic activity was low. (See §11.)

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and considerable solar corpuscular flux, which enveloped the earth for at least 7 h, did not produce either a clear DP or a clear DR. This corresponds to the apparent close relationship between the DP and the DR. It suggests that the DP and the DR cannot be independent. This supports our view of the significance of neutral line discharges, because the polar magnetic disturbances cannot be independent of the development of the ring current and so of the neutral line.

However, we do not know why such large differences can occur. Perhaps a rather uniform solar stream is relatively ineffective in producing either DP or DR. The production of auroral particles discussed by Parker (1958, 1959) and by Piddington (1960) may be criticized from this point of view. Differences between the consequences of apparently equally intense solar streams may also be due partly to some morphological differences in the solar streams, perhaps related to the injection mechanism of solar particles into the earth's field, and perhaps dependent on whether or not they carry a detached solar magnetic field DSM with them.

We would like to express our sincere thanks to Dr C. T. Elvey and our colleagues at the Geophysical Institute, College, Alaska, for many helpful discussions and for the use of their data. We are also greatly indebted to the Directors of the magnetic observatories who kindly sent us the magnetograms from which figures 13 and 16 are constructed.

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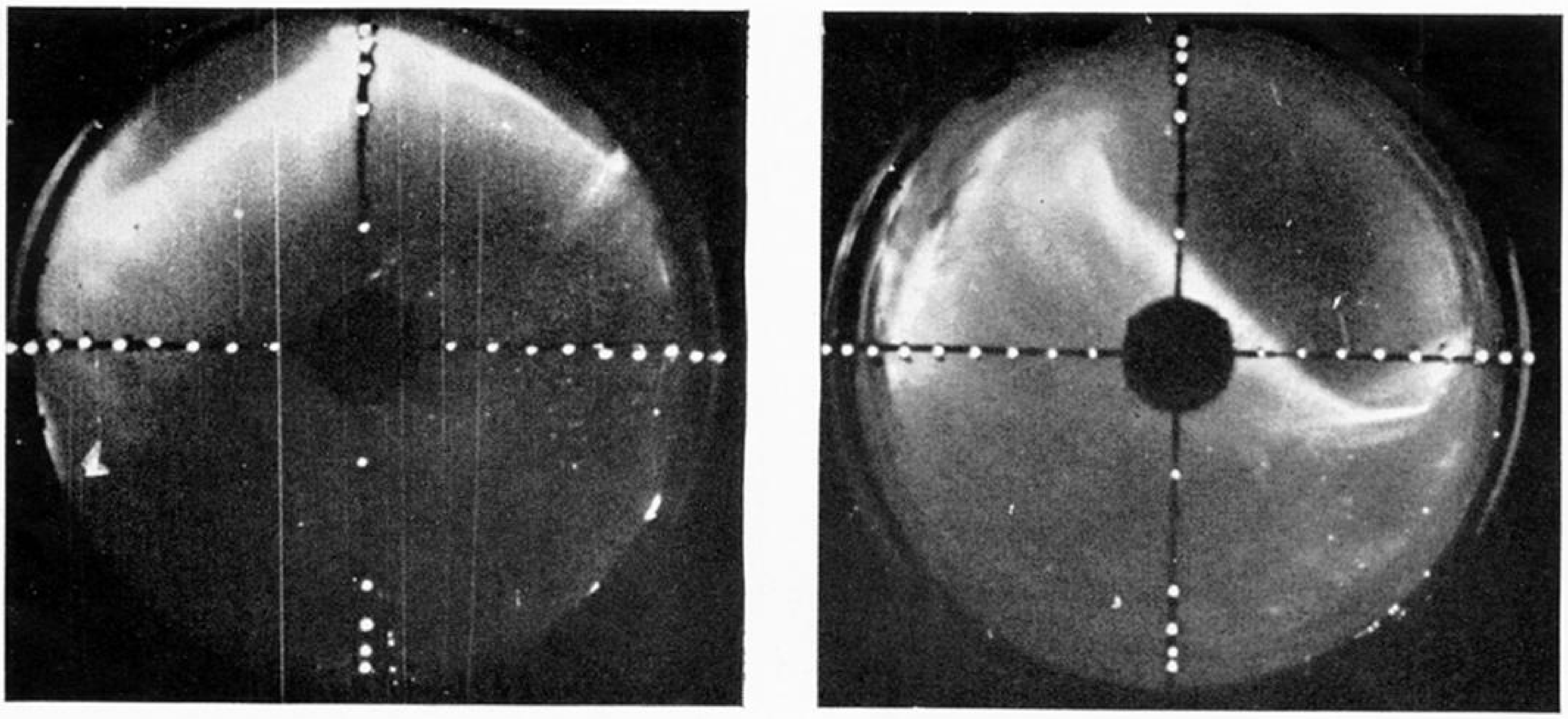
N N

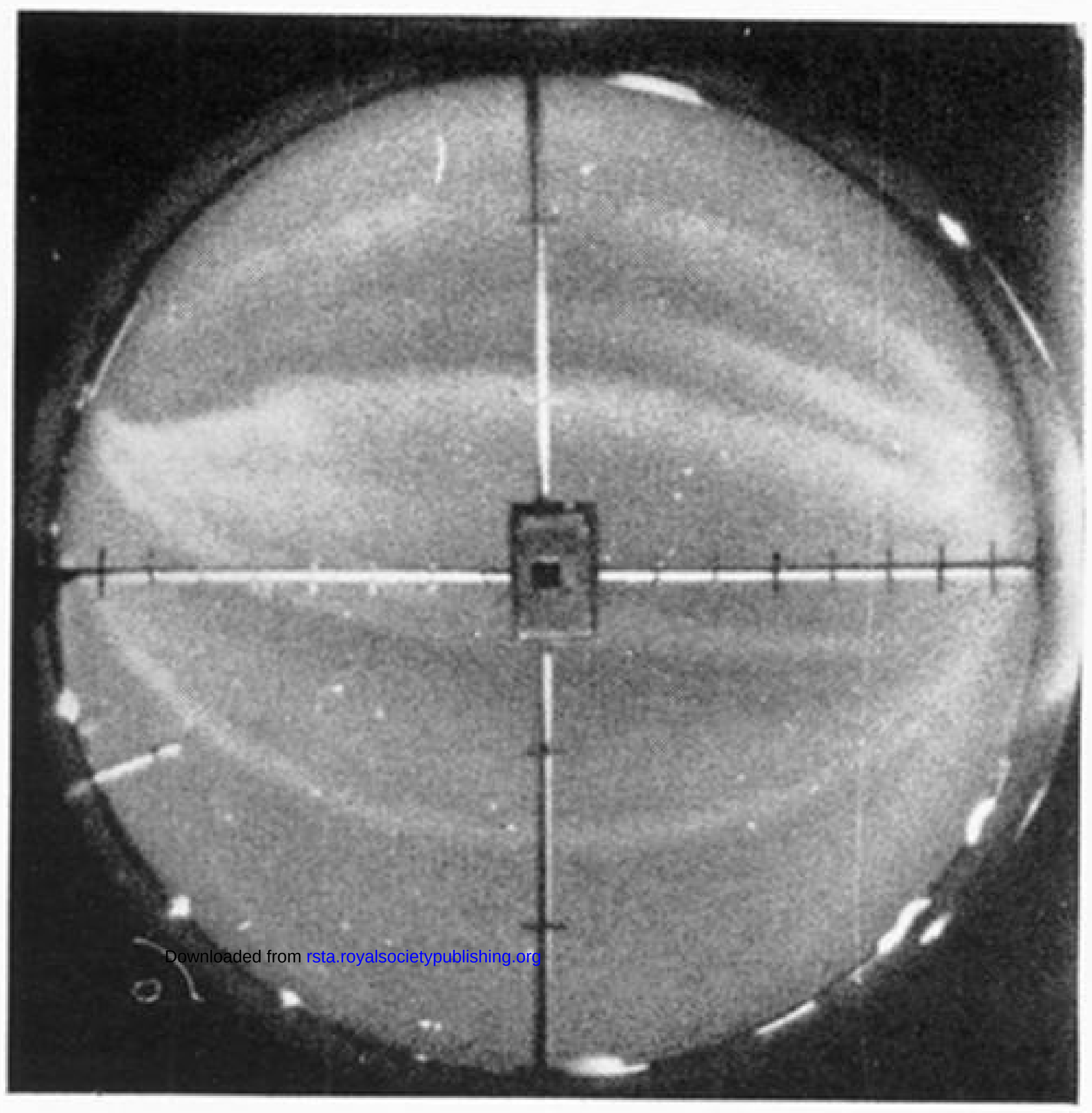


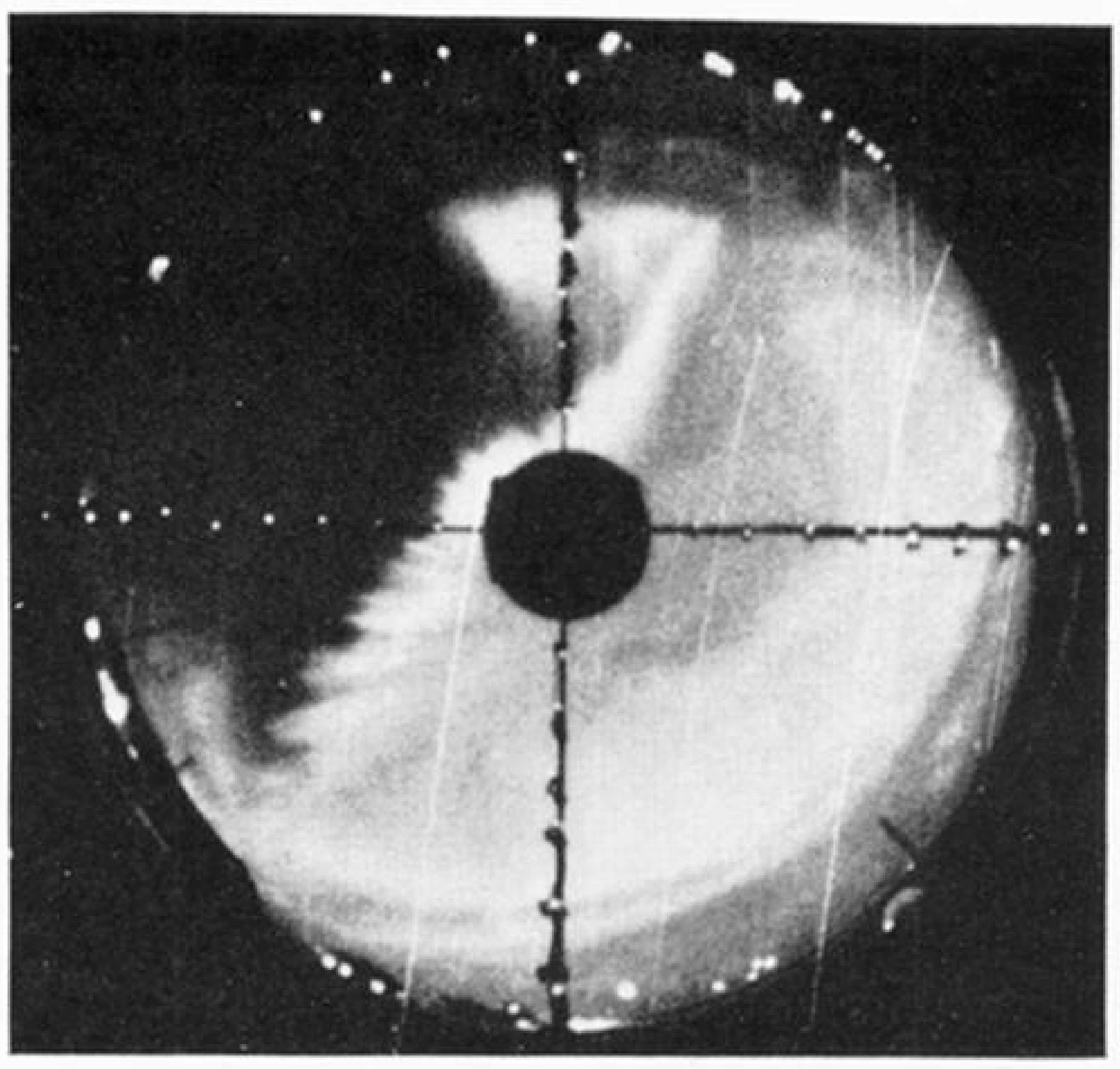
W E

(a) S S (b)

Figure 17

N N



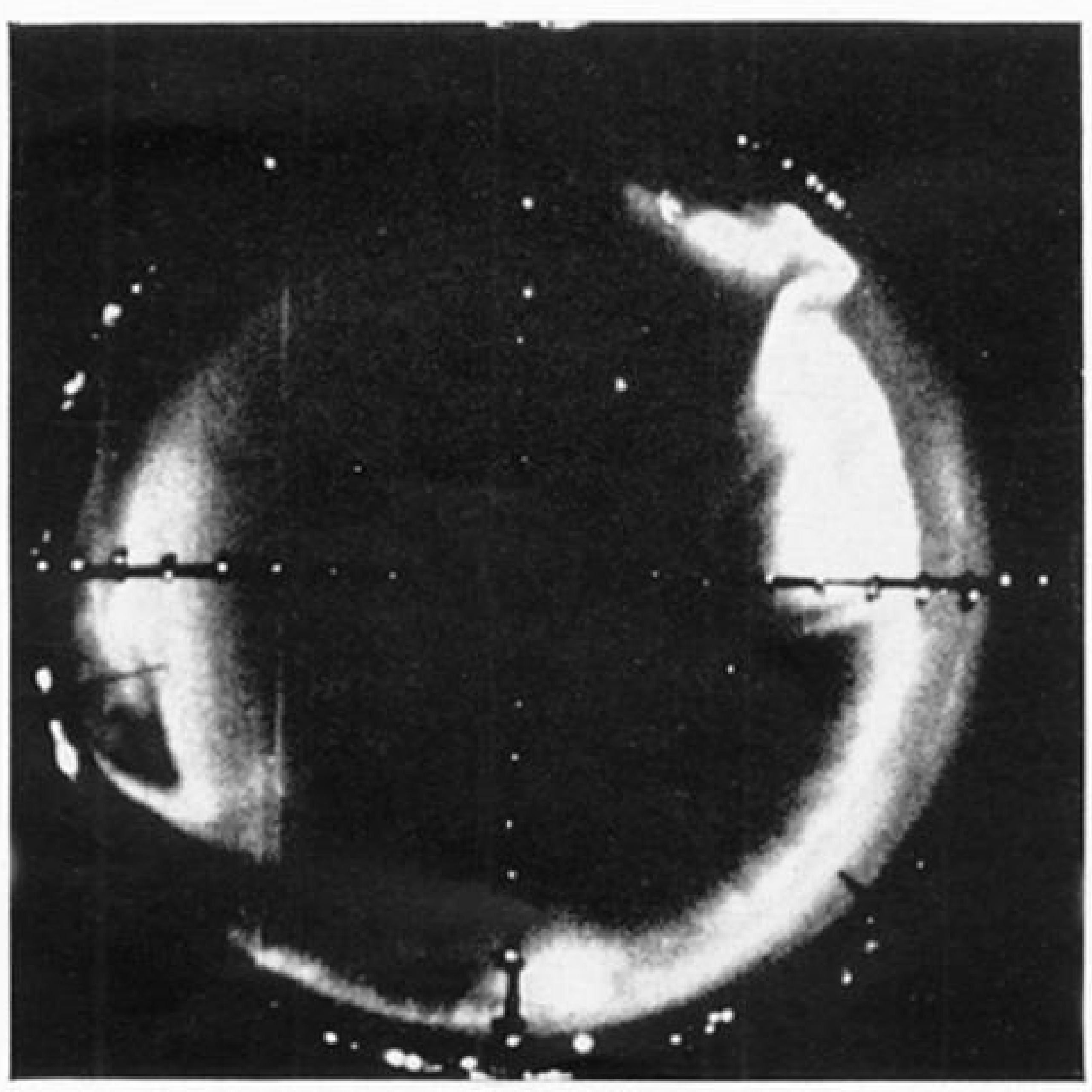


W E

S S

Figure 18 Figure 19

N



W E

S

Figure 24

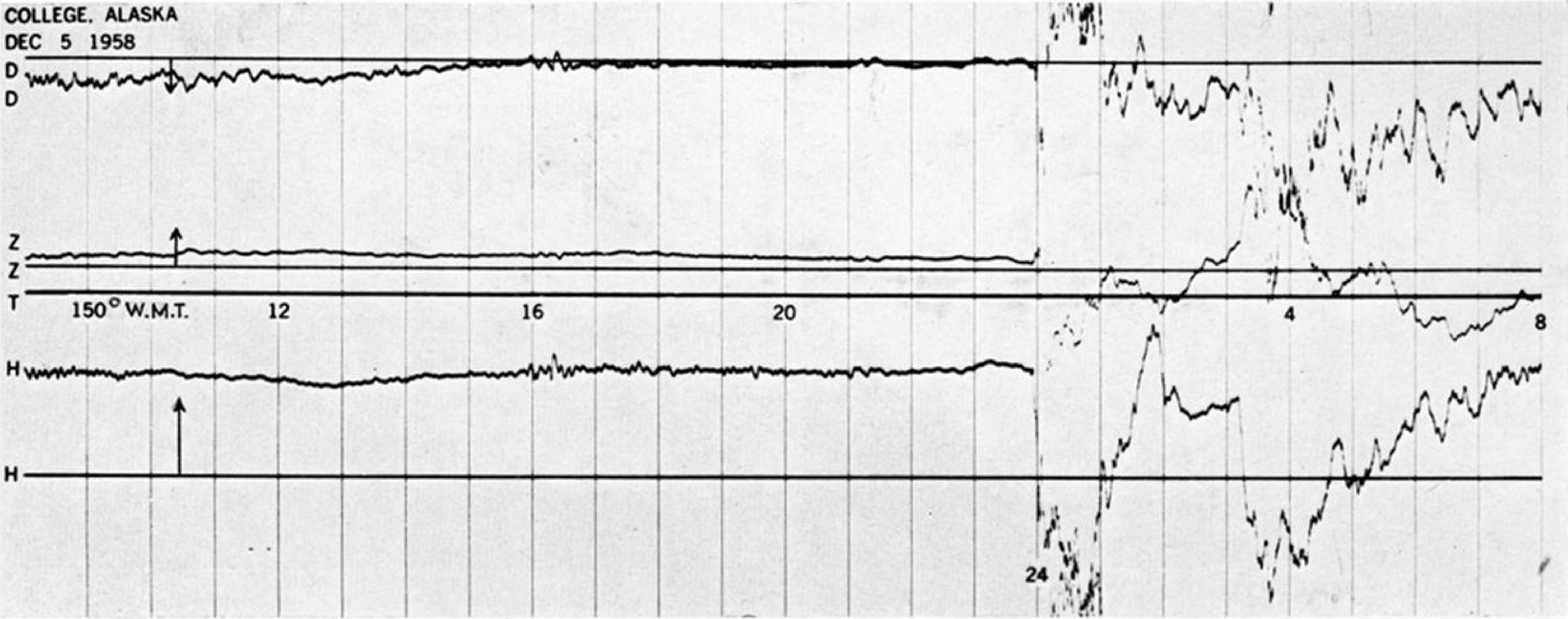


FIGURE 20. The College (Alaska) magnetogram of 5 to 6 December 1958. The hour marks refer to standard time of the meridian 150° W. Two small polar magnetic disturbances are shown, which began respectively at the local times 2357 and 0315. The arrows indicate the direction of increase of each component; their length corresponds to a variation of 100γ. The horizontal lines marked H, D, Z are the base lines from which the trace of the corresponding element is measured. (See § 9 (c).)

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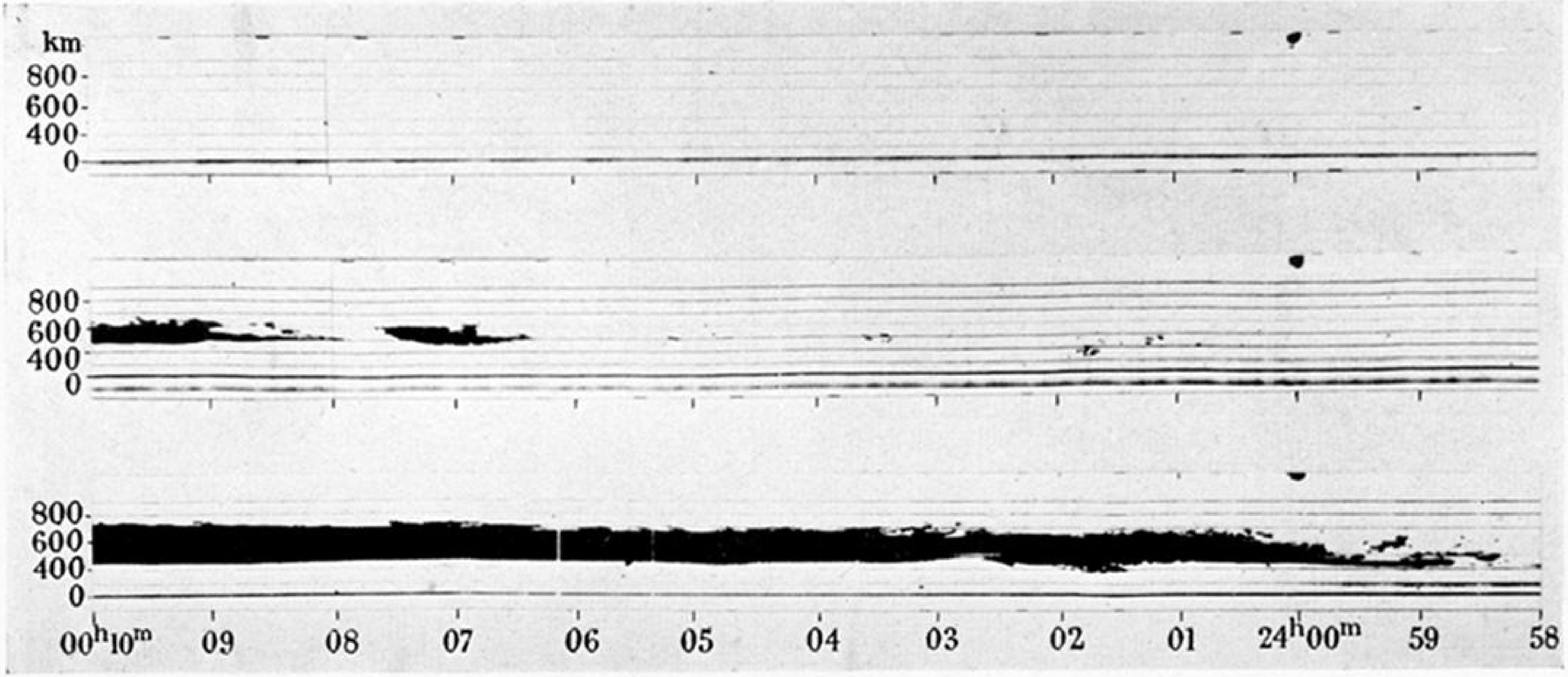

FIGURE 23. V.h.f. auroral radar data taken at Point Barrow (Alaska) (geomag. lat. 68·6° N), College (Alaska) (geomag. lat. 64·7° N) and Farewell (Alaska) (geomag. lat. 61·4° N) on the night of 5 to 6 December 1958. Note the radar echo at Farewell corresponding to the first polar disturbance in figure 20 (or the first break-up of the aurora in figure 21). (Leonard 1960, private communication). (See § 9 (*c*).)

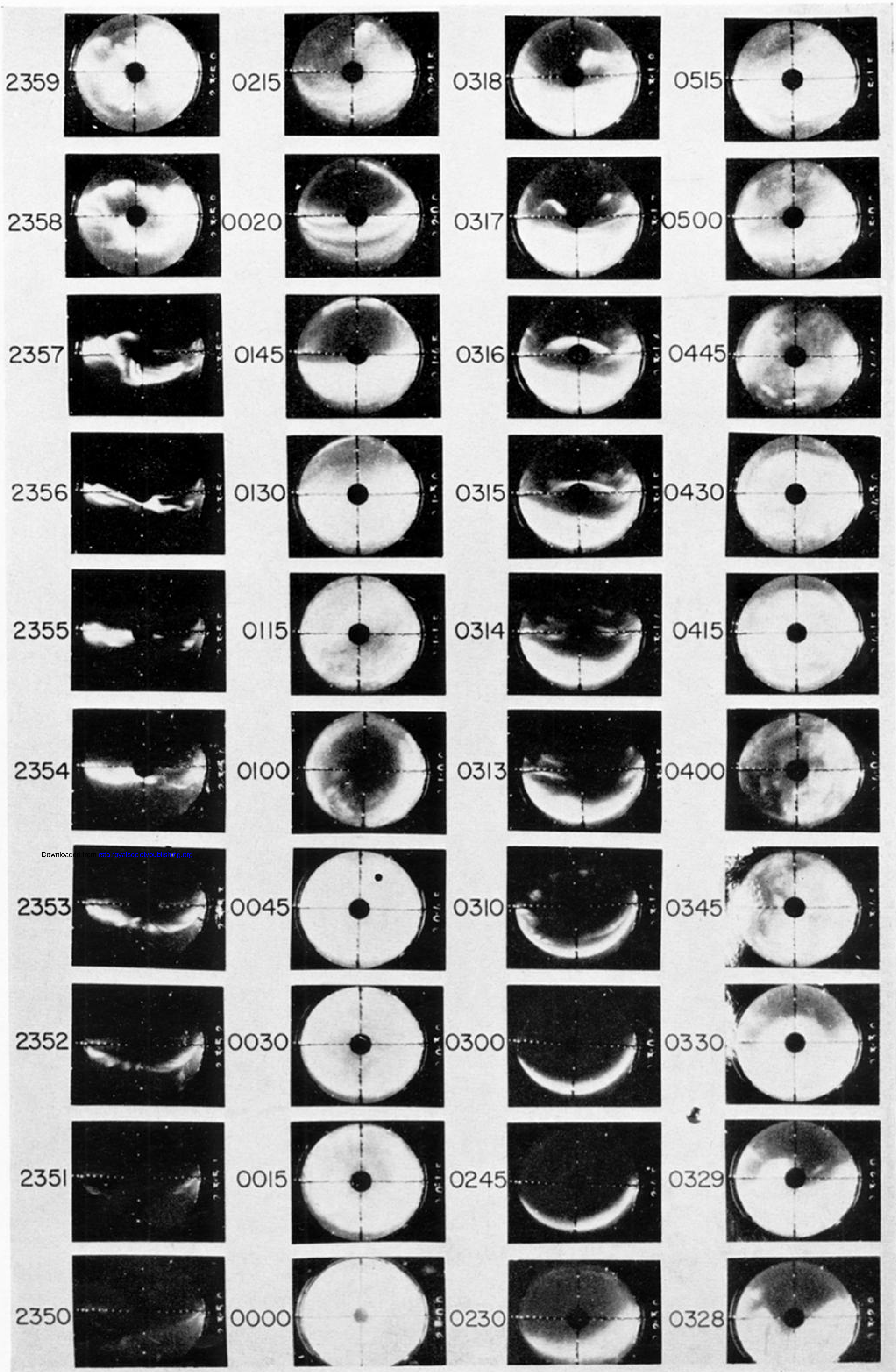

FIGURE 21. Some College (Alaska) all-sky camera photographs for the interval from 0950 to 1515 G.M.T. 6 December 1958 (local time, from 2350 5 to 0515 6 December 1958). This interval includes the two break-ups (at 2357 and 0315) associated with the two small polar magnetic disturbances shown in figure 20. (Geophysical Institute, Alaska.) (See § 9 (*c*).) In each photograph North is uppermost and East is on the right. The time marked 0020 should be 0200.

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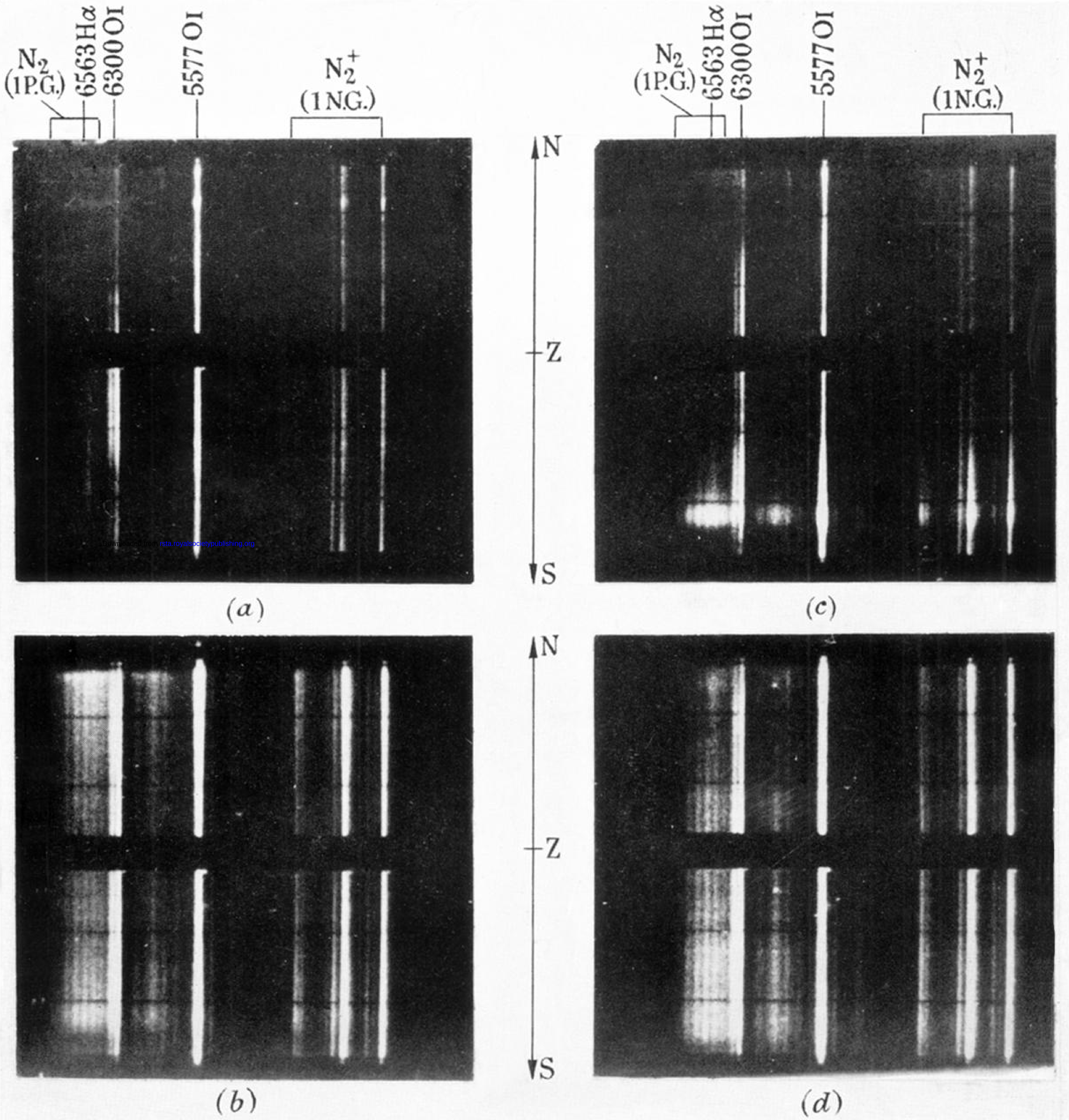

Figure 22. Four spectrograms taken at College, Alaska, on the night of 5 to 6 December 1958. Two of them, (*a*) and (*c*), were taken just before the auroral break-ups shown in figure 21. The others, (*b*) and (*d*), were taken just after the break-ups. (Rees 1960, private communication.) (See § 9 (*c*).)